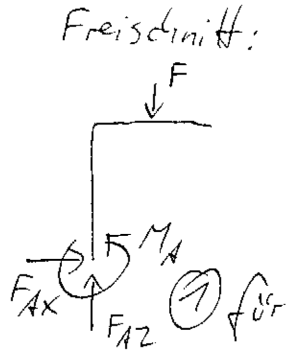
Musterlösung Ergänzungsfach, Klausur [illegible]

1

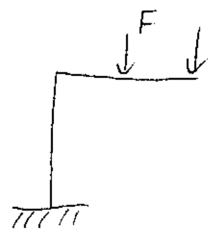
a) Freischnitt:



$$\left.\begin{aligned} \Sigma F_x = 0 &: \quad F_{Ax} = 0 \\ \Sigma F_y = 0 &: \quad F_{Az} = F \\ \Sigma M = 0 &: \quad M_A = F \cdot a \end{aligned}\right\} \ 1$$

1 für Freischnitt

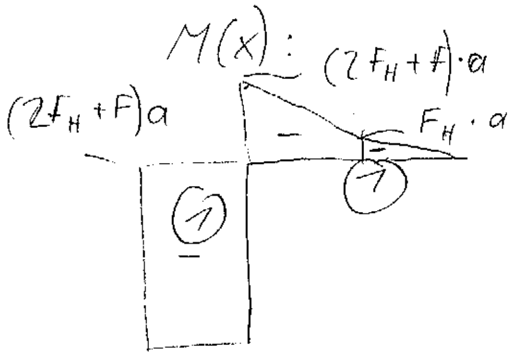
b)

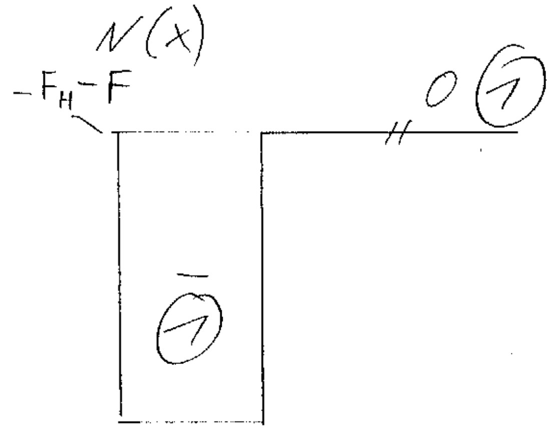


1 für Hilfskraft

$$W = \frac{1}{2EA}\int_0^l N^2(x)\,dx + \frac{1}{2EI}\int_0^l M^2(x)\,dx$$

1 für $N(x)$ & $M(x)$





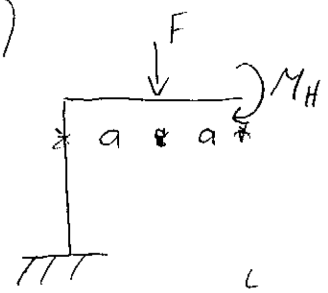
$$W = \frac{1}{2EA}\,3a\,(F_H+F)^2 + \frac{1}{2EI}\,3a\cdot\left[(2F_H+F)\cdot a\right]^2$$
$$+ \frac{1}{2EI}\,a\,\frac{1}{6}\Big(2\left[(2F_H+F)\cdot a\right]^2 + 2\left[F_H\cdot a\right]^2$$
$$+ 2\,(2F_H+F)\cdot a\cdot F_H\cdot a\Big)$$
$$+ \frac{1}{2EI}\,a\,\frac{1}{3}\,(F_H\cdot a)^2 \qquad 2$$

$$\delta = \left.\frac{\partial W}{\partial F_H}\right|_{F_H=0} = \frac{3aF}{EA} + \frac{41}{6}\frac{a^3F}{EI}$$

1 für Ableitung
1 für $F_H = 0$

$$\Rightarrow \quad F = \frac{5}{\frac{3a}{EA} + \frac{41}{6}\frac{a^3}{EI}} \qquad 1$$

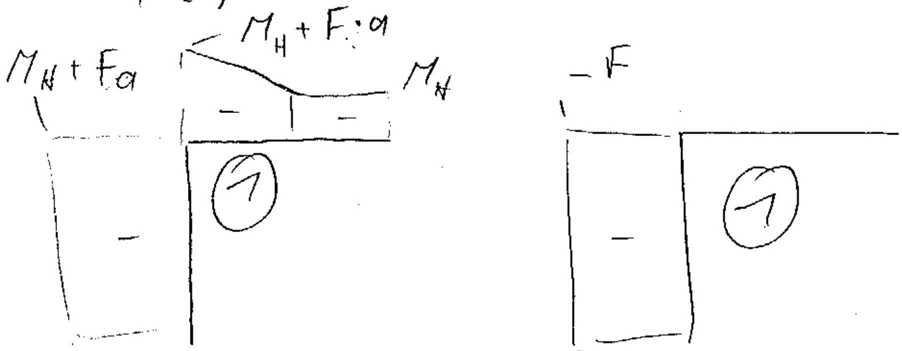
c)



1 für Hilfs

$$W = \frac{1}{2EA}\int_0^l N^2(x)\,dx + \frac{1}{2EI}\int_0^l M^2(x)\,dx$$

$M(x)$:



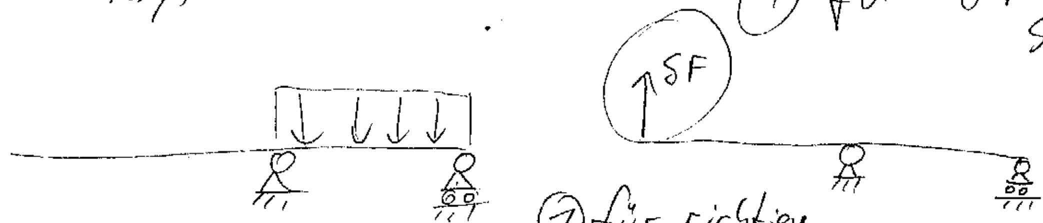
$$W = \frac{1}{2EA}\,3a\,F^2 + \frac{1}{2EI}\,3a\,(M_H + F\cdot a)^2$$
$$+ \frac{1}{2EI}\,a\,\frac{1}{6}\Big(2(M_H + F\cdot a)^2 + 2(M_H)^2 + 2(M_H + F\cdot a)M_H\Big)$$
$$+ \frac{1}{2EI}\,a\,M_H^2 \quad 1$$

$$\varphi = \left.\frac{\partial W}{\partial M_H}\right|_{M_H=0} = \frac{21\,Fa^2}{6EI} \quad 1$$
$$= \frac{7\,Fa^2}{3EI}$$

[2]

a) 4 Lagerkräfte
3 Gleichungen
$\Rightarrow$ 1-fach statisch unbestimmt 1

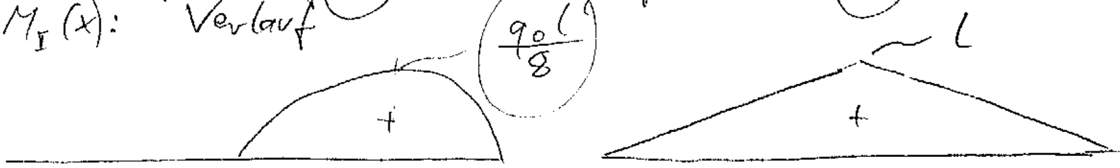
b) Teilsystem I: 1 für δF an richtigen Stelle



1 für richtige Amplitude

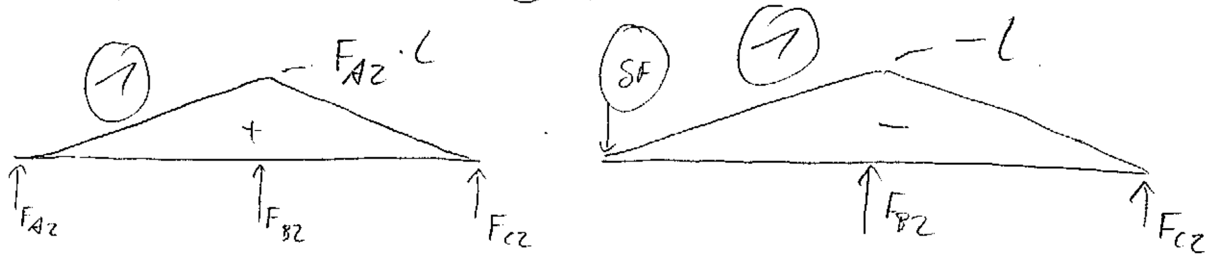
für richtigen Verlauf 1 1

$M_I(x)$:



$$\delta_I = \frac{1}{EI} l \frac{1}{3} \frac{q_0 l^2}{8} \cdot l = \frac{q_0 l^4}{EI \cdot 24}$$ 1

Teilsystem II: 1 für δF an richtiger Stelle



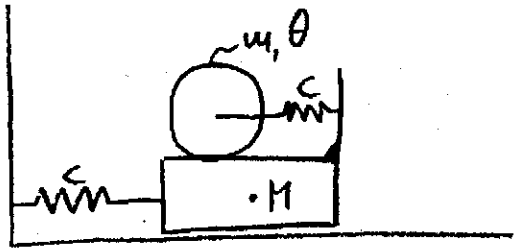
$$\delta_{II} = -\frac{1}{EI} 2l \frac{1}{3} F_{Az} l^2$$ 1

Superposition:

$$\delta_I - \delta_{II} = 0 \quad \Rightarrow \quad F_{Az} = -\frac{q_0 l}{16}$$

1 1

Aufgabe 3



1. Freiheitsgrade

$f = z - k = 6 - 4 = 2$

2 Körper à
3 Freiheitsgrade

Bedingungen:

$y_1 = 0 \qquad y_2 = 0$

$x_1 = x_s - r\varphi = x_w \qquad x_2 = q_2 = x_s$

$\varphi_1 = q_1 \qquad \varphi_2 = 0$ 1

2) Aufstellen der Energieen und generalisierte Kräfte

$$E^{kin} = \frac{1}{2} M \dot{x}_s^2 + \frac{1}{2} m \dot{x}_w^2 + \frac{1}{2} \theta_s \dot{\varphi}^2$$

$$= \frac{1}{2} M \dot{x}_s^2 + \frac{1}{2} m (\dot{x}_s - r\dot{\varphi})^2 + \frac{1}{2} \theta_s \dot{\varphi}^2$$ 1

$$U = \frac{1}{2} c (\Delta x_1)^2 + \frac{1}{2} c (\Delta x_2)^2 = \frac{1}{2} c r^2 \varphi^2 + \frac{1}{2} c x_s^2$$ 1

$$\Rightarrow L = \frac{1}{2} M \dot{x}_s^2 + \frac{1}{2} m (\dot{x}_s - r\dot{\varphi})^2 + \frac{1}{2} \theta_s \dot{\varphi}^2 - \frac{1}{2} c r^2 \varphi^2 - \frac{1}{2} c x_s^2$$

$\cancel{Q_i = 0}$

$$\left.\begin{aligned} Q_\varphi &= 0 \\ Q_x &= F(t) \end{aligned}\right\}$$ 1

3) Ableitungen der Lagrangeschen Gleichungen

Koordinate x_s

i) $\frac{\partial L}{\partial \dot{x}_s} = M\dot{x}_s + m(\dot{x}_s - r\dot{\varphi})$

ii) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_s}\right) = M\ddot{x}_s + m\ddot{x}_s - rm\ddot{\varphi} = (M+m)\ddot{x}_s - rm\ddot{\varphi}$ 1

iii) $\frac{\partial L}{\partial x_s} = -cx_s$ 1

Koordinate φ

i) $\frac{\partial L}{\partial \dot{\varphi}} = m(\dot{x}_s - r\dot{\varphi})(-r) + \Theta_s\dot{\varphi}$

ii) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = -mr\ddot{x}_s + mr^2\ddot{\varphi} + \Theta_s\ddot{\varphi}$ 1

iii) $\frac{\partial L}{\partial \varphi} = -cr\varphi^2$ 1

4) Einsetzen in Lagrange Gleichungen

Koordinate x_s

$(M+m)\ddot{x}_s - rm\ddot{\varphi} + cx_s = \cancel{0}\ F(t)$ 1

Koordinate φ

$-mr\ddot{x}_s + mr^2\ddot{\varphi}\,\Theta_s\ddot{\varphi} + cr\varphi^2 = 0$ 1