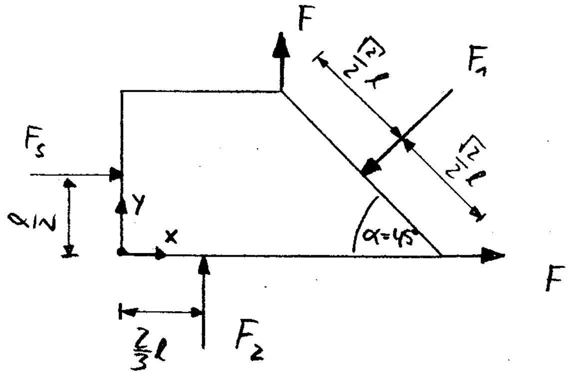


Bei der Bestimmung der Angriffspunkte der Resultierenden F_s, F_1, F_2 kann von der Symmetrie der Streckenlast bzw. dem Schwerpunkt des Dreieckes Gebrauch gemacht werden. Die Kräfte F_s, F_1 und F_2 greifen wie skizziert an der Scheibe an. Die Kräfte selbst erhält man aus

$$F_1 = q_1 \frac{l}{\cos\alpha} = q_1 \sqrt{2}\, l$$

$$F_2 = \frac{1}{2} q_2\, 2l = q_2\, l$$

$$F_s = \int_0^l q_s(y)\,dy = \int_0^l \frac{q_0}{2} \sin\left(\frac{\pi}{l} y\right) dy = \frac{q_0 l}{\pi}$$

Resultierende Kraft

$$\Sigma F_x = F + F_s - F_1 \cos\alpha = F + \frac{q_0 l}{\pi} - \sqrt{2}\, q_1 l \cdot \frac{\sqrt{2}}{2}$$

$$\Sigma F_x = F + \frac{q_0 l}{\pi} - q_1 l = 0 \qquad (1)$$

$$\Sigma F_y = F + F_2 - F_1 \sin\alpha = F + q_2 l - \sqrt{2}\, q_1 l \frac{\sqrt{2}}{2}$$

$$\Sigma F_y = F + q_2 l - q_1 l = 0 \qquad (2)$$

Das resultierende Moment, bezogen auf Punkt 0

$$\Sigma M^{(0)} = F_1 \cos\alpha \frac{l}{2} - F_1 \sin\alpha \frac{3}{2} l + F_2 \frac{2}{3} l - F_s \frac{l}{2} + F l = 0$$

$$\Sigma M^{(0)} = \frac{q_1 l^2}{2} - \frac{3}{2} q_1 l^2 + \frac{2}{3} q_2 l^2 - \frac{q_0 l^2}{2\pi} + F l = 0 \qquad (3)$$

Gl. 1 – 3 nach q_0, q_1 und q_2 auflösen

aus 1) $q_0 = \pi\left(q_1 - \frac{F}{l}\right)$

aus 2) $q_2 = q_1 - \frac{F}{l}$

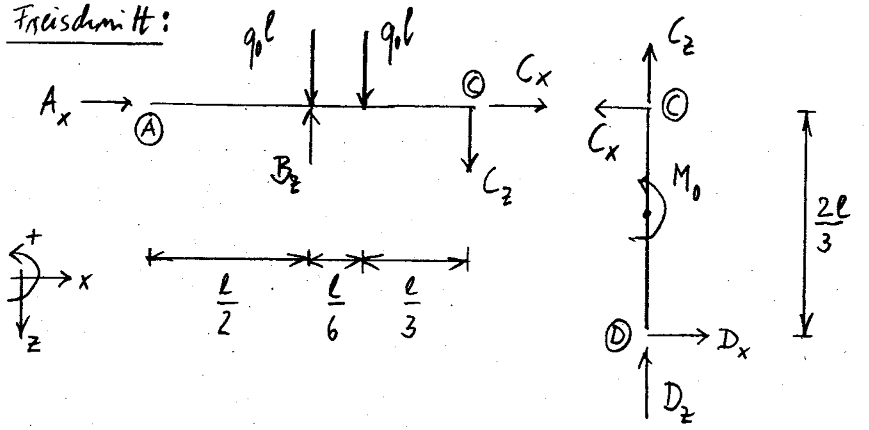
ergibt mit Gleichung 3)

$$-q_1 l^2 + \frac{2}{3}\left(q_1 - \frac{F}{l}\right) l^2 - \pi\left(q_1 - \frac{F}{l}\right)\frac{l^2}{2\pi} + F l = 0$$

$$q_1\left\{-1 + \frac{2}{3} - \frac{1}{2}\right\} + \frac{F}{l}\left\{-\frac{2}{3} + \frac{1}{2} + 1\right\} = 0$$

$\underline{\underline{q_1 = \frac{F}{l}}}$ $\qquad$ $\underline{q_0 = 0}$ $\qquad$ $\underline{q_2 = 0}$

Freischnitt:



Gleichgewicht:

linker Teil

$$\sum M_{(B)} = -\frac{\ell}{6} q_0 \ell - C_z \frac{\ell}{2} \overset{!}{=} 0 \quad \rightarrow \quad C_z = -q_0 \frac{\ell}{3}$$

$$\sum M_{(C)} = -B_z \frac{\ell}{2} + q_0 \ell \left(\frac{\ell}{2} + \frac{\ell}{3} \right) \overset{!}{=} 0 \quad \rightarrow \quad B_z = \frac{5}{3} q_0 \ell$$

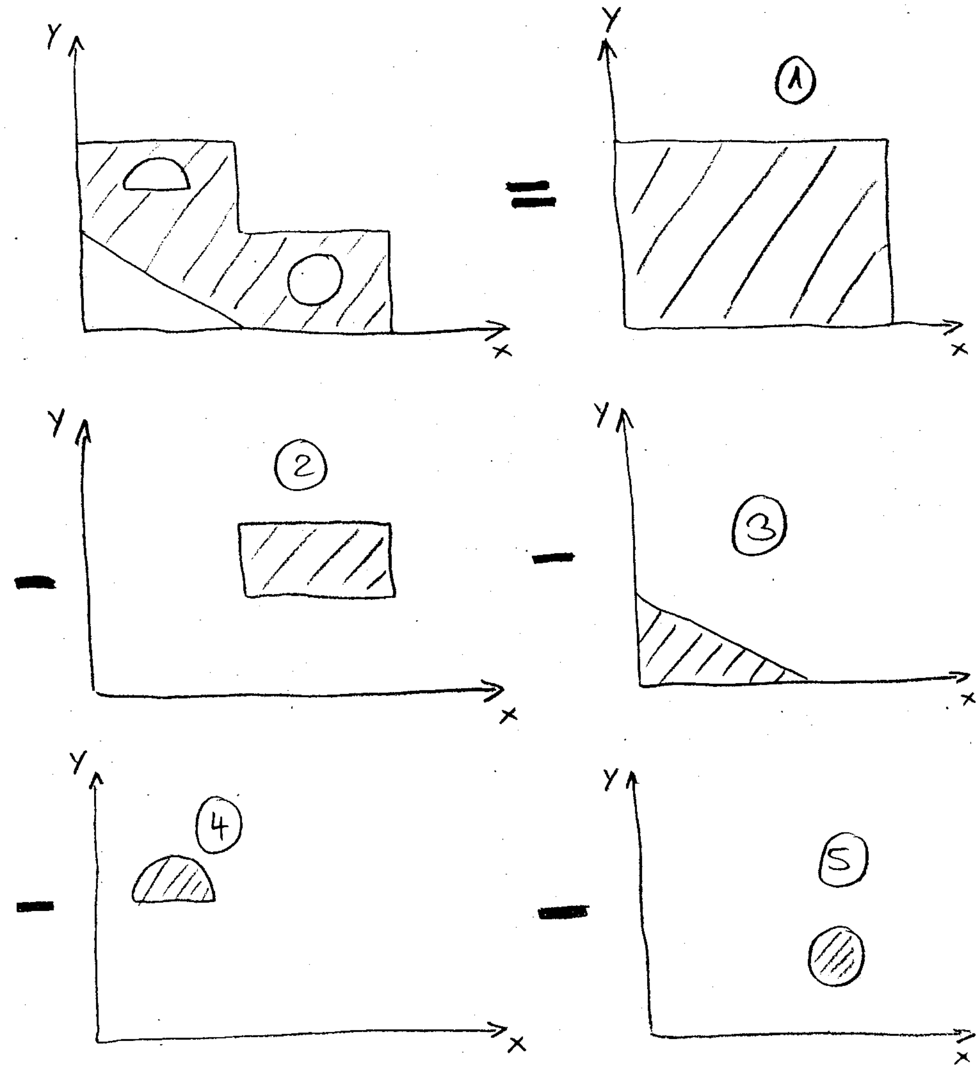
$$\sum F_x = A_x + C_x \overset{!}{=} 0 \quad \rightarrow \quad A_x = -C_x = \frac{3}{2} \frac{M_0}{\ell}$$

rechter Teil

$$\sum M_{(C)} = M_0 + D_x \frac{2}{3} \ell \overset{!}{=} 0 \quad \rightarrow \quad D_x = -\frac{3}{2} \frac{M_0}{\ell}$$

$$\sum F_x = D_x - C_x \overset{!}{=} 0 \quad \rightarrow \quad C_x = -\frac{3}{2} \frac{M_0}{\ell}$$

$$\sum F_z = D_z + C_z \overset{!}{=} 0 \quad \rightarrow \quad D_z = q_0 \frac{\ell}{3}$$



i	A_i	x_{si}	y_{si}	$x_{si}A_i$	$y_{si}A_i$
1	bh	$\frac{b}{2}$	$\frac{h}{2}$	$\frac{b^2h}{2}$	$\frac{bh^2}{2}$
2	$-\frac{b}{2}\cdot\frac{h}{2}$	$\frac{3b}{4}$	$\frac{3h}{4}$	$-\frac{3}{16}b^2h$	$-\frac{3}{16}bh^2$
3	$-\frac{1}{2}\cdot\frac{b}{2}\cdot\frac{h}{2}$	$\frac{1}{3}\cdot\frac{b}{2}$	$\frac{1}{3}\cdot\frac{h}{2}$	$-\frac{1}{48}b^2h$	$-\frac{1}{48}bh^2$
4	$-\frac{1}{2}\left(\frac{d}{2}\right)^2\pi$	$\frac{b}{4}$	$\frac{3h}{4}+\frac{4}{3\pi}\cdot\frac{d}{2}$	$-\frac{\pi}{32}bd^2$	$-\frac{\pi}{8}d^2\left(\frac{3h}{4}+\frac{2d}{3\pi}\right)$
5	$-\left(\frac{d}{2}\right)^2\pi$	$\frac{3b}{4}$	$\frac{h}{4}$	$-\frac{3\pi}{16}bd^2$	$-\frac{\pi}{16}d^2h$

$$x_s=\frac{\sum_{i=1}^{5}x_{si}A_i}{\sum_{i=1}^{5}A_i}=\frac{\frac{24b^2h}{48}-\frac{9b^2h}{48}-\frac{1}{48}b^2h-\frac{\pi}{32}bd^2-\frac{6\pi}{32}bd^2}{bh-\frac{bh}{4}-\frac{bh}{8}-\frac{\pi d^2}{8}-\frac{\pi d^2}{4}}$$

$$=\frac{\frac{14}{48}b^2h-\frac{7\pi}{32}bd^2}{\frac{5}{8}bh-\frac{3\pi}{8}d^2}=\frac{\frac{14}{6}b^2h-\frac{7\pi}{4}bd^2}{5bh-3\pi d^2}=7b\,\frac{\frac{2}{6}bh-\frac{\pi}{4}d^2}{5bh-3\pi d^2}$$

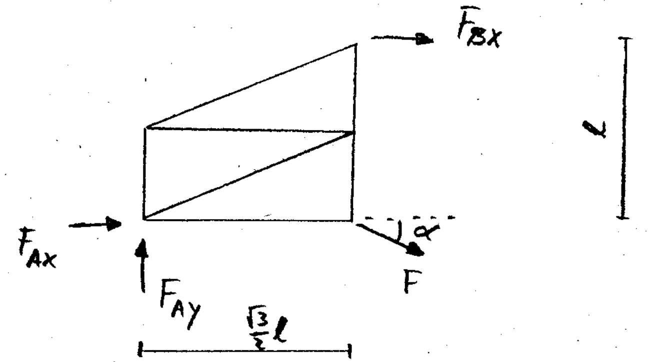
$$=\frac{7b}{12}\cdot\frac{4bh-3\pi d^2}{5bh-3\pi d^2}$$

$$y_s=\frac{\sum_{i=1}^{5}y_{si}A_i}{\sum_{i=1}^{5}A_i}=\frac{\frac{24bh^2}{48}-\frac{9bh^2}{48}-\frac{bh^2}{48}-\frac{3\pi}{32}d^2h-\frac{d^3}{12}-\frac{\pi}{16}d^2h}{\frac{5}{8}bh-\frac{3\pi}{8}d^2}$$

$$=\frac{\frac{14}{48}bh^2-\frac{5\pi}{32}d^2h-\frac{d^3}{12}}{5bh-3\pi d^2}\cdot 8=\frac{\frac{14}{6}bh^2-\frac{5\pi}{4}d^2h-\frac{2d^3}{3}}{5bh-3\pi d^2}$$

$$=\frac{1}{12}\cdot\frac{28bh^2-15\pi d^2h-8d^3}{5bh-3\pi d^2}$$

Auflagerreaktionen



$$\Sigma F_x = F_{Ax} + F_{Bx} + F\cos\alpha = 0$$

$$\Sigma F_x = F_{Ax} + F_{Bx} + \frac{\sqrt{3}}{2} F = 0 \qquad (1)$$

$$\Sigma F_y = F_{Ay} - F\sin\alpha = 0 \qquad (2)$$

$$\underline{\underline{F_{Ay} = \frac{F}{2}}}$$

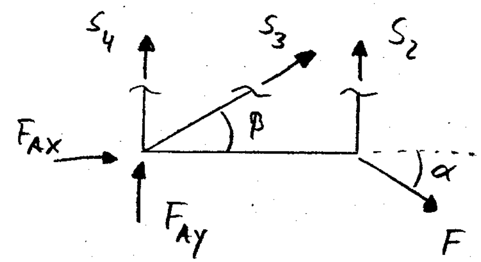
$$\Sigma M^{(A)} = -F \cdot \sin\alpha \, \frac{\sqrt{3}}{2}\ell - F_{Bx}\,\ell = 0 \qquad (3)$$

$$\underline{\underline{F_{Bx} = -\frac{\sqrt{3}}{4} F}}$$

aus (1) folgt dann

$$F_{Ax} = -F_{Bx} - \frac{\sqrt{3}}{2} F = \frac{\sqrt{3}}{4} F - \frac{\sqrt{3}}{2} F = \underline{\underline{-\frac{\sqrt{3}}{4} F = F_{Ax}}}$$

Ritterschnitt durch die Stäbe 2, 3 und 4



$$\tan\beta = \frac{\ell/2}{\ell\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

$$\beta = 30^\circ = \alpha$$

$$\Sigma F_x = F_{Ax} + F\cos\alpha + S_3\cos\beta = 0$$

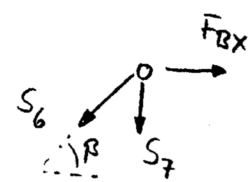
$$= -\frac{\sqrt{3}}{4} F + \frac{\sqrt{3}}{2} F + S_3 \frac{\sqrt{3}}{2} = 0 \Rightarrow \underline{\underline{S_3 = -\frac{F}{2}}}$$

$$\Sigma M^{(A)} = -F\sin\alpha \, \frac{\sqrt{3}}{2}\ell + S_2 \frac{\sqrt{3}}{2}\ell = 0 \Rightarrow \underline{\underline{S_2 = F\sin\alpha = \frac{F}{2}}}$$

$$\Sigma F_y = F_{Ay} - F\sin\alpha + S_4 + S_3\sin\beta + S_2 = 0$$

$$S_4 = -\frac{F}{2} + \frac{F}{2} + \frac{F}{2}\,\frac{1}{2} - \frac{F}{2} = \underline{\underline{-\frac{F}{4} = S_4}}$$

Knotenschnitt bei B



$$\Sigma F_x = F_{Bx} - S_6\cos\beta = 0$$

$$\Sigma F_y = -S_7 - S_6\sin\beta = 0$$

$$\underline{\underline{S_6 = -\frac{F}{2}}} \qquad \underline{\underline{S_7 = +\frac{F}{4}}}$$