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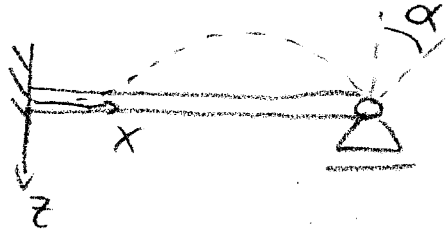

$q(x) = 0$

DGL für die Biegelinie

$$EI\, w^{IV}(x) = 0$$

$$EI\, w'''(x) = C_1$$

$$EI\, w''(x) = C_1 x + C_2$$

$$EI\, w'(x) = \frac{1}{2} C_1 x^2 + C_2 x + C_3$$

$$EI\, w(x) = \frac{1}{6} C_1 x^3 + \frac{1}{2} C_2 x^2 + C_3 x + C_4$$

Randbedingungen

$x = 0$

1) $w(0) = 0$ 2) $w'(0) = 0$

$x = \ell$

3) $w(\ell) = 0$ 4) $w'(\ell) = \alpha$

Bestimmen des Konstanten

1) $w(0) = 0 \Rightarrow C_4 = 0$

2) $w'(0) = 0 \Rightarrow C_3 = 0$

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3) $w(\ell) = \frac{1}{6} c_1 \ell^3 + \frac{1}{2} c_2 \ell^2 = 0 \Rightarrow c_2 = -\frac{1}{3} c_1 \ell$

4) $w'(\ell) = \alpha \Rightarrow \frac{c_1}{2} \ell^2 + c_2 \ell = EI\alpha$ mit c_2 folgt

$$\frac{c_1}{2} \ell^2 - \frac{1}{3} c_1 \ell^2 = \alpha EI \Rightarrow c_1 = 6\alpha \frac{EI}{\ell^2}$$

und $$c_2 = -2\alpha \frac{EI}{\ell}$$

Biegelinie $w(x) = \alpha \ell \left\{ \left(\frac{x}{\ell}\right)^3 - \left(\frac{x}{\ell}\right)^2 \right\}$

Einspannmoment und Querkraft bei $x = 0$

$$M(0) = -EI\, w''(0) = -c_2 = 2\alpha \frac{EI}{\ell}$$

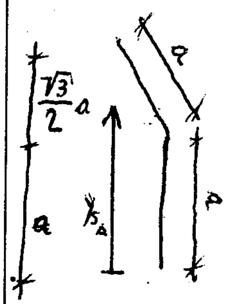
$$Q(0) = -EI\, w'''(0) = -c_1 = -6\alpha \frac{EI}{\ell^2}$$

„Zusammenbau"

$\square + \triangle - \bigcirc$

$$A_\triangle = \frac{1}{2} a \Big(\underbrace{a \frac{\sqrt{3}}{2}}_{\sin 60^\circ}\Big) = \frac{\sqrt{3}}{4} a^2 = 0{,}433\, a^2$$

$$A_\bigcirc = \frac{\pi d^2}{4} = \pi \frac{\left(\frac{a}{4}\right)^2}{4} = \frac{\pi}{64} a^2 = 0{,}0491\, a^2$$



$$y_{S_\triangle} = a + \frac{1}{3} a \frac{\sqrt{3}}{2} = a\left(1 + \frac{\sqrt{3}}{6}\right) = \frac{a}{6}\left(6 + \sqrt{3}\right)$$
$$= 1{,}289\, a$$

$$I_{zz_\triangle} = \frac{a}{36}\left(\frac{\sqrt{3}}{2} a\right)^3 = \frac{\sqrt{3}}{96} a^4 = 0{,}018\, a^4$$

$$I_{zz_\bigcirc} = \frac{\pi}{64}\left(\frac{a}{4}\right)^4 = 1{,}917 \cdot 10^{-4} a^4 = 0{,}0002\, a^4$$

$$I_{zz_\square} = \frac{a^4}{12} = 0{,}083\, a^4$$

	A_i	y_i	S_i
+ □	a^2	$0{,}5a$	$0{,}5\,a^3$
+ Δ	$0{,}433a^2$	$1{,}289a$	$0{,}558\,a^3$
− O	$-0{,}0491a^2$	a	$-0{,}0491$
	$1{,}3839a^2$		$1{,}0089a^3$

$$\hat{y}_s = \frac{\sum S_i}{\sum A_i} = \frac{1{,}0089\,a^3}{1{,}3839\,a^2} = \underline{\underline{0{,}729a}} \qquad \hat{z}_s = 0$$

	$\lvert y_s - y_i\rvert$	I_{zz}	$(y_s-y_i)^2 A_i$
+ □	$0{,}229a$	$0{,}0833a^4$	$0{,}0524\,a^4$
+ Δ	$0{,}560a$	$0{,}0180\,a^4$	$0{,}1357\,a^4$
− O	$0{,}271\,a$	$-0{,}0002\,a^4$	$-0{,}0036\,a^4$
		$0{,}1011\,a^4$	$0{,}1845\,a^4$

$$I_{zz} = \sum\left[I_{zz} + (y_{s_i} - y_i)^2 A_i\right] \simeq \underline{\underline{0{,}2856\,a^4}}$$

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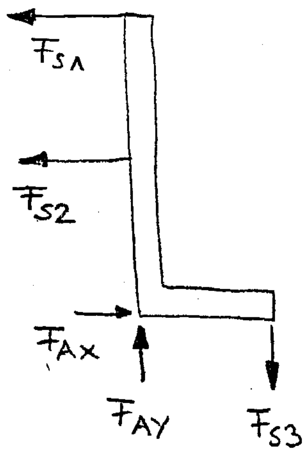
a) Freischnitt



Gleichgewichtsbedingungen

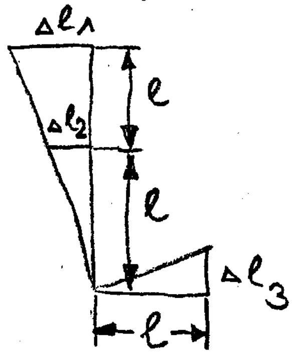
$\rightarrow$ $F_{Ax} - F_{S1} - F_{S2} = 0$

$\uparrow$ $F_{Ay} - F_{S3} = 0$

$\curvearrowleft A$ $F_{S1} \cdot 2l + F_{S2} \cdot l - F_{S3} \cdot l = 0$ (1)

b) Anordnung 2-fach statisch unbestimmt

Verformungsbedingungen



$\Delta l_1 = 2 \Delta l_2$ (2)

$\Delta l_2 = - \Delta l_3$ (3)

c) Hooke

Stab 1: $\sigma_1 = E\varepsilon_1$, $\frac{F_{S1}}{A} = E\frac{\Delta l_1}{l}$, $F_{S1} = \frac{EA}{l}\Delta l_1$ (4)

Stab 2: $F_{S2} = \frac{EA}{l}\Delta l_2$ (5)

Stab 3: $\varepsilon_3 = \varepsilon_{elast.3} + \varepsilon_{therm3}$

$$\frac{\Delta l_3}{l} = \frac{\sigma_3}{E} + \alpha\Delta T \quad , \quad \Delta l_3 = \frac{F_{S3}\, l}{EA} + \alpha\Delta T\, l \qquad (6)$$

F_{S2} berechnen

(4) und (5) in (2):

$$\frac{F_{S1}\, l}{EA} = 2\cdot\frac{F_{S2}\, l}{EA} \quad , \quad F_{S1} = 2F_{S2} \qquad (7)$$

(5) und (6) in (3):

$$\frac{F_{S2}\, l}{EA} = -\frac{F_{S3}\, l}{EA} - \alpha\Delta T l \quad , \quad F_{S2} = -F_{S3} - EA\alpha\Delta T \qquad (8)$$

(7) und (8) in (1):

$$2F_{S2}\cdot 2l + F_{S2}\cdot l + (F_{S2} + EA\alpha\Delta T)\cdot l = 0$$

$$6F_{S2} + EA\alpha\Delta T = 0 \quad , \quad \boxed{F_{S2} = -\frac{EA\alpha\Delta T}{6}} \quad \text{(Druck)}$$

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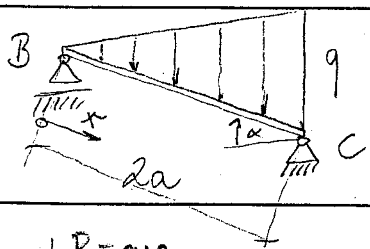

Lösungen zum Aufgabenkatalog

Aufgabe 4

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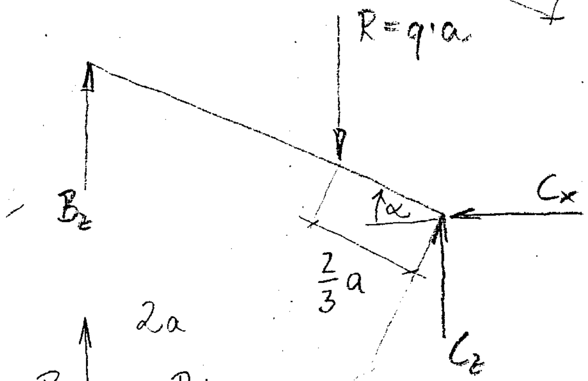

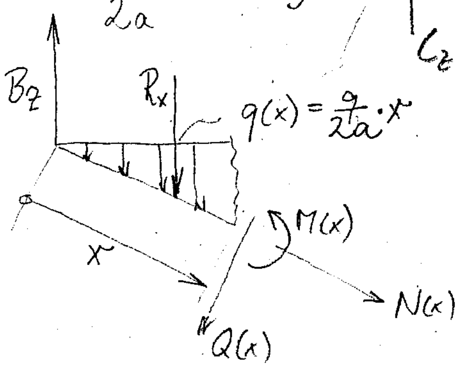

(1) $\underline{\underline{C_x = 0}}$

(2) $B_z + C_z = q \cdot a$

(3) $R \cdot \frac{2}{3} a \cdot \cos\alpha = B_z \cdot 2a \cos\alpha$

(3) $\rightsquigarrow$ $\underline{\underline{B_z = \frac{1}{3} qa}}$ in (2)

$\underline{\underline{C_z = \frac{2}{3} qa}}$

$$R_x = \int_{\tilde{x}=0}^{\tilde{x}=x} q(\tilde{x})\, d\tilde{x} = \int_0^x \frac{q}{2a} \cdot \tilde{x}\, d\tilde{x}$$

$$\underline{\underline{R_x = \frac{q}{4a} x^2}}$$

$$N(x) = (-R_x + B_z) \sin\alpha$$

$$= \left[-\frac{1}{4} q \frac{x^2}{a} + \frac{1}{3} qa\right] \sin\alpha$$

$$\underline{\underline{N(x) \overset{\alpha=30^\circ}{=} \frac{1}{2} qa \left[\frac{1}{3} - \left(\frac{x}{2a}\right)^2\right]}}$$

$$Q(x) = (-R_x + B_z) \cos\alpha$$

$$\underline{\underline{Q(x) \overset{\alpha=30^\circ}{=} \frac{\sqrt{3}}{2} qa \left[\frac{1}{3} - \left(\frac{x}{2a}\right)^2\right]}}$$

$$M(x) = \left[B_z \cdot x - R_x \cdot \frac{x}{3}\right] \cos\alpha$$

$$= \left[\frac{1}{3} qax - \frac{1}{4} q \frac{x^3}{3a}\right] \cos\alpha$$

$$\underline{\underline{M(x) \overset{\alpha=30^\circ}{=} \frac{\sqrt{3}}{3} qa^2 \left[\frac{x}{2a} - \left(\frac{x}{2a}\right)^3\right]}}$$