Musterlösung

$\boxed{1}$

$$m(t) \cdot \frac{dv}{dt} = \mu v_r - m(t) \cdot g \qquad \mu = 2\,\frac{kg}{s}$$

$$\frac{dv}{dt} = \frac{\mu v_r}{m_0 - \mu t} - g \qquad V_r \text{ wird } \underline{\text{entgegen}} \text{ } v \text{ positiv gezählt}$$

$$v = - v_r \ln\left(1 - \frac{\mu}{m_0} t\right) - g \cdot t + \underbrace{\cancel{v_0}}_{=0 \text{ für } v(t=0)=0}$$

$$v(t=t^*) = 1802{,}2\,\frac{m}{s} - 1697{,}1\,\frac{m}{s}$$

$$\underline{\underline{v(t^*) = 105{,}1\,\frac{m}{s}}}$$

$$m(t^*) = m_0 - \mu \cdot t^*$$

$$\underline{\underline{m(t^*) = 54\,kg}}$$

Da v_r entgegen v positiv gezählt wird, gilt ab $t = t^*$:

$$\underline{v_r = -900\,\frac{m}{s}}$$

$$v(t = 183s) = v(t^*) - v_r \ln\left(1 - \frac{\mu}{m(t^*)} \cdot 10s\right) - g \cdot 10s$$

$$= (105{,}1 - 416{,}4 - 98{,}1)\,\frac{m}{s}$$

$$\underline{\underline{v(t = 183s) = -409{,}4\,\frac{m}{s}}}$$

$$m(t = 183s) = (54 - 20)\,kg$$

$$\underline{\underline{= 34\,kg}}$$

$\boxed{2}$ a) $v_1 = \dot{x}_M + r\dot{\varphi}$ $\qquad \dot{v}_1 = \ddot{x}_M + r\ddot{\varphi}$

$v_2 = \text{const} = -\dot{x}_M + r\dot{\varphi}$ $\qquad \dot{v}_2 = 0 = -\ddot{x}_M + r\ddot{\varphi}$

$$\left.\begin{matrix} \dot{v}_1 = \ddot{x}_M + r\ddot{\varphi} \\ \dot{v}_2 = 0 = -\ddot{x}_M + r\ddot{\varphi} \end{matrix}\right\} \rightsquigarrow \underline{\underline{\ddot{x}_M = \frac{1}{2}\dot{v}_1}} \quad \underline{\underline{\ddot{\varphi} = \frac{1}{2}\frac{\dot{v}_1}{r}}}$$

$v_1 - v_2 = 2\dot{x}_M$

b) $m\ddot{x}_M = -S_1 - S_2 + mg$ $\qquad$ I

$\Theta\ddot{\varphi} = (S_2 - S_1)\,r$ $\qquad$ II

$$\left.\begin{matrix} \ddot{x}_M \text{ in I}: & \frac{1}{2}m\dot{v}_1 = -S_1 - S_2 + mg \\ \ddot{\varphi} \text{ in II} & \frac{1}{2}\Theta\frac{\dot{v}_1}{r} = (S_2 - S_1)\,r \end{matrix}\right\} \rightsquigarrow \begin{matrix} \underline{\underline{S_1 = \frac{mg}{2} - \dot{v}_1\left(\frac{m}{4} + \frac{\Theta}{4r^2}\right)}} \\ \underline{\underline{S_2 = \frac{mg}{2} - \dot{v}_1\left(\frac{m}{4} - \frac{\Theta}{4r^2}\right)}} \end{matrix}$$

c) $\Theta = mr^2$

d) $\underline{\underline{S_2 = \frac{mg}{2}}}$

$\underline{\underline{S_1 = \frac{mg}{2} - \frac{m}{2}\dot{v}_1}}$

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a) $\Theta_{zz}^{S} = \varrho \cdot d \cdot \int\limits_{\varphi=0}^{\pi} \int\limits_{r=R-t}^{R+t} r^2(\cos^2\varphi + \sin^2\varphi)\, r\, dr\, d\varphi$

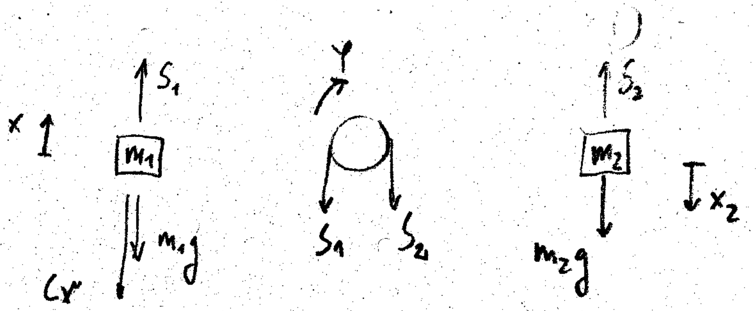
$= \varrho \cdot \pi \cdot d \cdot \frac{1}{4}\left\{(R+t)^4 - (R-t)^4\right\}$

$\underline{\underline{\Theta_{zz}^{S} = 2\pi \varrho d\,(R^3 t + R t^3)}}$

neu: b) $\Theta_{zz}^{SP} = \Theta_{zz}^{S} - y_{SP}^2\, m$ mit $m = \varrho \cdot d \cdot \pi \cdot R\, 2 \cdot t$

$\underline{\underline{\Theta_{zz}^{SP} = 2\pi \varrho d\,(R^3 t + R t^3 - y_{SP}^2 R t)}}$

c) $\underline{\underline{\Theta_{zz}^{SM} = \Theta_{zz}^{S} + M \cdot R^2}}$



Kinematik

$\ddot{x} = \ddot{x}_2$

$\ddot{\varphi} = \frac{\ddot{x}}{a}$

$$m_1\ddot{x} = -m_1 g + S_1 - cx$$

$$\Theta^S \cdot \frac{\ddot{x}}{a} = (S_2 - S_1)\cdot a$$

$$m_2\ddot{x} = m_2 g - S_2$$

Einsetzen:

$$S_1 = m_2(g - \ddot{x}) - \Theta^S \ddot{x}$$

$$m_1\ddot{x} = -m_1 g + m_2 g - \left(\frac{\Theta^S}{a^2} + m_2\right)\ddot{x}$$

$$\ddot{x} + \underbrace{\frac{c}{m_1 + m_2 + \frac{\Theta^S}{a^2}}}_{\omega_0^2} x = \underbrace{\frac{g(m_1 - m_2)}{m_1 + m_2 + \frac{\Theta^S}{a^2}}}_{F}$$

Partik. Lösung:

$$\underline{\underline{x_p(t) = \frac{F}{\omega_0^2} = x_{stat}}}$$

Allg. Lösung

$$x(t) = A \cdot C_{\omega_0 t} + B S_{\omega_0 t} + \frac{F}{\omega_0^2}$$

$\cos(\cdot) =: C_{(\cdot)}$

$\sin(\cdot) =: S_{(\cdot)}$

Anpassen an AB:

$$x(t=0) = A + \frac{F}{\omega_0^2} \overset{!}{=} 0 \quad \Rightarrow A = -\frac{F}{\omega_0^2}$$

$$\dot{x}(t=0) = B\omega_0 \overset{!}{=} 0 \quad \Rightarrow B = 0$$

Spezielle Lösung $\underline{\underline{x(t) = -\frac{F}{\omega_0^2} C_{\omega_0 t} + \frac{F}{\omega_0^2}}}$

$$\underline{\underline{T = \frac{2\pi}{\omega_0}}}$$