

2a)

$$\left. \begin{aligned} w(0) = 0 & \quad a_0 = 0 \\ w'(0) = 0 & \quad a_1 = 0 \end{aligned} \right\} \textcircled{1}$$

$$w(l) = 0 \quad a_2 l^2 + a_3 l^3 = 0 \quad \Rightarrow a_2 \stackrel{\textcircled{1}}{=} -a_3 \cdot l$$

$$\begin{aligned} w(x) &= a_2 x^2 + a_3 x^3 \\ &= -a_3 l x^2 + a_3 x^3 \end{aligned}$$

$$w(x) \stackrel{\textcircled{1}}{=} a_3 \cdot \frac{(x^3 - lx^2)}{l^2}$$

$$w'(x) = a_3 (3x^2 - 2lx)$$

$$w'' = a_3 (6x - 2l)$$

$$\begin{aligned} \text{b) } W &= \frac{EJ}{2} \int_0^l w''^2 dx + \frac{1}{2} c_M w(l)^2 \quad \textcircled{1} \\ &= \frac{EJ}{2} a_3^2 \int_0^l (6x - 2l)^2 dx + \frac{1}{2} c_M a_3^2 (3l^2 - 2l^2)^2 \\ &= \frac{EJ}{2} a_3^2 \left[\frac{1}{3} \cdot 6 \cdot (6x - 2l)^3 \right]_0^l + \frac{1}{2} c_M a_3^2 l^4 \\ &= \frac{EJ}{36} a_3^2 \cdot (64l^3 + 8l^3) + \frac{1}{2} c_M a_3^2 l^4 \end{aligned}$$

$$= 2EJ a_3^2 l^3 + \frac{1}{2} c_M a_3^2 l^4$$

$$W \stackrel{\textcircled{1}}{=} a_3^2 (2EJ l^3 + \frac{1}{2} c_M l^4)$$

$$A = \int_0^l q_0 \cdot w(x) dx = q_0 \cdot a_3 \int_0^l (x^3 - lx^2) dx$$

$$= q_0 a_3 \cdot \left(\frac{1}{4} l^4 - \frac{1}{3} l^4 \right) = -\frac{1}{12} q_0 a_3 l^4 \quad \textcircled{1} \quad (\text{Endergebnis})$$

$$\text{c) } \delta(W-A) = \frac{\partial(W-A)}{\partial a_3} \delta a_3 \stackrel{\textcircled{1}}{=} 0 \quad \Leftrightarrow \quad a_3 (4EJ l^3 + c_M l^4) + \frac{1}{12} q_0 l^4 = 0$$

$$a_3 = -\frac{1}{12} \frac{q_0 \cdot l}{4EJ + c_M \cdot l}$$

$$w(x) = -\frac{1}{12} \frac{q_0 l}{4EJ + c_M l} \cdot (x^3 - lx^2) \quad \textcircled{1} \quad (\text{Endergebnis})$$