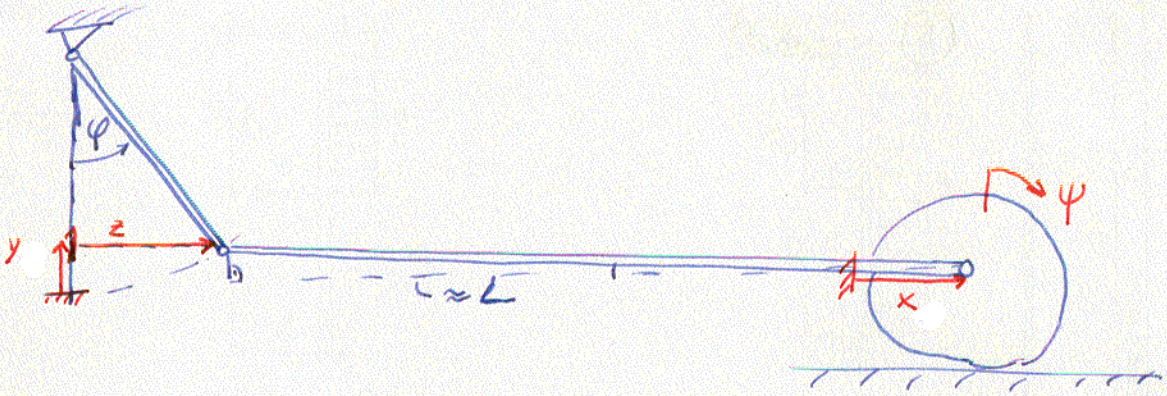
Aufgabe 4.)

a.)



b.)

$$y = l - l\cdot\cos\varphi = l(1-\cos\varphi) \quad (1)$$

$$x = z = l\cdot\sin\varphi \quad (1) \qquad \Rightarrow \dot{x} = l\cdot\dot{\varphi}\cos\varphi$$

$$\psi = \frac{x}{r} = \frac{l}{r}\cdot\sin\varphi \quad (1) \qquad \Rightarrow \dot{\psi} = \frac{l}{r}\cdot\dot{\varphi}\cos\varphi \quad \} (1)$$

$$E_{pot} = m_1\cdot g\cdot y = m_1 g\, l(1-\cos\varphi) \quad (1)$$

$$E_{kin} = \frac{1}{2} m_1\cdot(l\cdot\dot{\varphi})^2 + \frac{1}{2} m_2\cdot\dot{x}^2 + \frac{1}{2}\Theta\,\dot{\psi}^2 \quad (1)$$

$$= \frac{1}{2}\left(m_1\cdot l^2\dot{\varphi}^2 + m_2\, l^2\cos^2\varphi\,\dot{\varphi}^2 + \Theta\left(\frac{l}{r}\right)^2\cos^2\varphi\,\dot{\varphi}^2\right)$$

$$\Rightarrow L = E_{kin} - E_{pot} = \frac{1}{2} l^2\left(m_1 + m_2\cdot\cos^2\varphi + \frac{\Theta}{r^2}\cos^2\varphi\right)\dot{\varphi}^2 - m_1\cdot g\cdot l(1-\cos\varphi) \quad (1)$$

c.)

$$\frac{\partial L}{\partial\dot{\varphi}} = l^2\left(m_1 + m_2\cos^2\varphi + \frac{\Theta}{r^2}\cos^2\varphi\right)\dot{\varphi} \quad (1)$$

$$\frac{d}{dt}\cos^2\varphi = -\dot{\varphi}\sin\varphi\, 2\cos\varphi$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\varphi}}\right) = l^2\left(m_1 + m_2\cos^2\varphi + \frac{\Theta}{r^2}\cos^2\varphi\right)\ddot{\varphi} + l^2\left(m_2 + \frac{\Theta}{r^2}\right)(-)\,\dot{\varphi}\sin\varphi\, 2\cos\varphi\,\dot{\varphi}$$

$$= l^2\left(m_1 + m_2\cos^2\varphi + \frac{\Theta}{r^2}\cos^2\varphi\right)\ddot{\varphi} - l^2\left(m_2 + \frac{\Theta}{r^2}\right)2\cdot\sin\varphi\cos\varphi\,\dot{\varphi}^2 \quad (1)$$

$$\frac{\partial L}{\partial\varphi} = -\dot{\varphi}^2 l^2\left(m_2 + \frac{\Theta}{r^2}\right)\cdot\cos\varphi\cdot\sin\varphi - m_1\cdot g\cdot l\cdot\sin\varphi \quad (1)$$

$$\Rightarrow l^2\left(m_1 + \left(m_2 + \frac{\Theta}{r^2}\right)\cos^2\varphi\right)\ddot{\varphi} - l^2\, 2\left(m_2 + \frac{\Theta}{r^2}\right)\sin\varphi\cos\varphi\,\dot{\varphi}^2 + \dot{\varphi}^2 l^2\left(m_2 + \frac{\Theta}{r^2}\right)\cos\varphi\sin\varphi + m_1 g l\cdot\sin\varphi$$

$$(1) = 0$$

$$l^2\left(m_1 + \left(m_2 + \frac{\Theta}{r^2}\right)\cos^2\varphi\right)\ddot{\varphi} - l^2\dot{\varphi}^2\left(m_2 + \frac{\Theta}{r^2}\right)\sin\varphi\cos\varphi + m_1 g l\sin\varphi = 0$$