

NuLö 1 (11 Punkte = $\frac{1}{2}$ Semester)

a)
$$[\underline{F}] = 1 \frac{\text{N}}{\text{kg}} = 1 \frac{\text{m}}{\text{s}^2} \quad \left[\frac{1}{\rho} \text{grad } p \right] = [\text{grad } U] = \frac{\text{m}^3}{\text{kg}} \frac{1}{\text{m}^3}$$

Probe: ①

b) $U = g \cdot x_3 \quad \Leftrightarrow \underline{f} = -\text{grad } U = -g \underline{e}_3 = +g$

c) $\frac{1}{\rho} \frac{\partial p}{\partial x_1} = 0, \quad \frac{1}{\rho} \frac{\partial p}{\partial x_2} = 0, \quad \frac{1}{\rho} \frac{\partial p}{\partial x_3} = -g \quad \text{①}$

$\frac{1}{\rho} \frac{dp}{dx_3} = -g$ ①

d)
$$\int_{x_3=0}^{x_3=H} \frac{1}{\rho} d\tilde{p} = \int -g dx_3 \quad \text{①}$$

 $\tilde{p} = p(x_3=0) \quad x_3=0$

NR.: $\frac{p}{\rho} = \frac{p_0}{\rho_0}$

$\frac{1}{\rho} = \frac{p_0}{p} \cdot \frac{1}{\rho_0}$

$\frac{1}{\rho} = \left(\frac{p_0}{p} \right)^{\frac{1}{\alpha}} \cdot \frac{1}{\rho_0} \quad \text{①}$

$$\frac{\rho}{\rho_0} \frac{p_0^{\frac{1}{\alpha}}}{\rho_0^{\frac{1}{\alpha}}} \left[\tilde{p}^{-\frac{\alpha-1}{\alpha}} \right]_{p_0}^{p(H)} = -g \cdot H \quad \text{①} + \text{①} \quad \text{richtig integriert links + rechts}$$

$$p(H)^{\frac{\alpha-1}{\alpha}} = p_0^{\frac{\alpha-1}{\alpha}} - \frac{\alpha-1}{\alpha} \frac{\rho_0 g H}{\rho_0^{\frac{1}{\alpha}}}$$

$$p(H)^{\frac{\alpha-1}{\alpha}} = p_0^{\frac{\alpha-1}{\alpha}} \left(1 - \frac{\alpha-1}{\alpha} \frac{\rho_0 g H}{p_0} \right)$$

① Produkt statt Differenz

$$p(H) = p_0 \left(1 - \frac{\alpha-1}{\alpha} \cdot \frac{\rho_0 g H}{p_0} \right)^{\frac{\alpha}{\alpha-1}}$$

① bei völlig richtigem Endergebnis