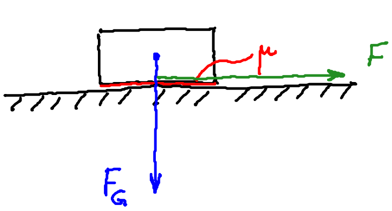
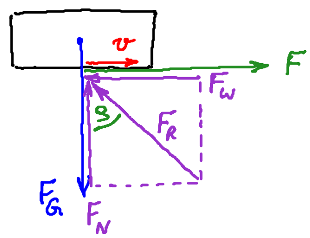
Vorlesung am 19.4.2018

- Wiederholung Coulombsches Reibungsgesetz
- Anwendungen

  Schraube, Seilreibung und Riementriebe, Seilbremsen, Keil

Wiederholung





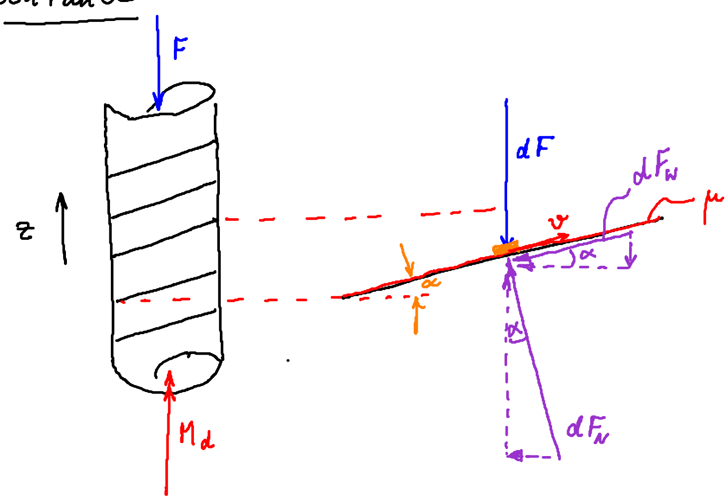
Coulombgesetz:

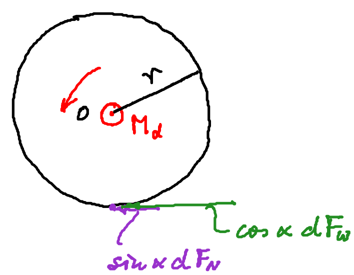
$$F_W = \mu F_N$$

$$\tan\varrho = \frac{F_W}{F_N} = \mu$$

$$\Rightarrow \varrho = \arctan\mu$$

Schraube





$$\Sigma F_z = 0 : \quad -dF + dF_N \cos\alpha - dF_W \sin\alpha = 0 \qquad \Big| \int \text{Schraubenwicklung}$$

$$-F + \cos\alpha \, F_N - \sin\alpha \, F_W = 0$$

$$\Rightarrow F_N [\cos\alpha - \mu \sin\alpha] = F \Rightarrow F_N = \frac{F}{\cos\alpha - \mu\sin\alpha}$$

$$\Sigma M^{(0)} = 0 : \quad -r\, dF_N \sin\alpha - r\, dF_W \cos\alpha + dM_d = 0 \qquad \Big| \int \text{Schraubenwicklung}$$

$$-r\sin\alpha \, F_N - r\cos\alpha \, F_W + M_d = 0$$

Coulomb: $F_W = \mu F_N$

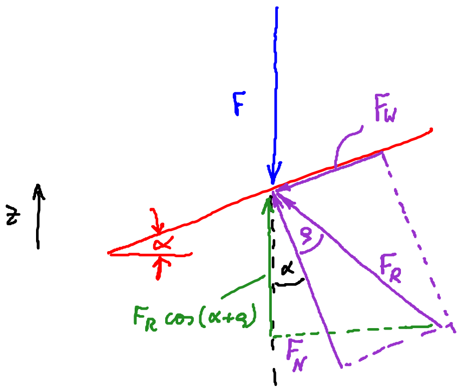
$$\Rightarrow M_d = r[\sin\alpha \, F_N + \cos\alpha \, F_W] = r[\sin\alpha + \mu\cos\alpha] F_N$$

$$= r F \frac{\sin\alpha + \mu\cos\alpha}{\cos\alpha - \mu\sin\alpha}$$

$M_d \to \infty$ für $\cos\alpha - \mu\sin\alpha = 0$

$$\Rightarrow \mu = \frac{\cos\alpha}{\sin\alpha}$$

Rechnung mit Reibungswinkel ϱ



$$\tan\varrho = \frac{F_W}{F_N} \qquad F_W = \mu F_N \quad \Rightarrow \tan\varrho = \mu = \frac{\sin\varrho}{\cos\varrho}$$

$$\Sigma F_z = 0 : \quad -F + F_R\cos(\alpha+\varrho) = 0$$

$$F_R = \frac{F}{\cos(\alpha+\varrho)} \Rightarrow F_N = F\frac{\cos\varrho}{\cos(\alpha+\varrho)} = ①$$

$$F_N = F_R\cos\varrho$$

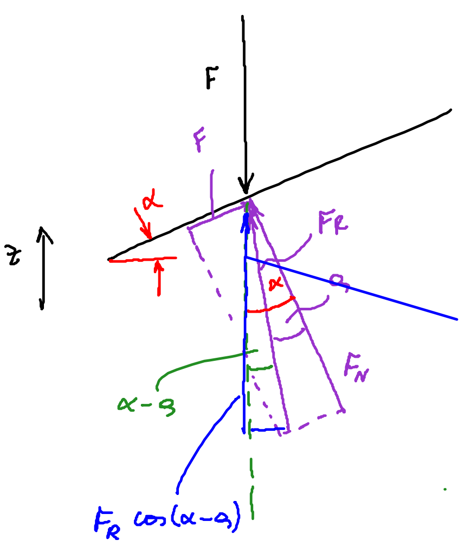
$$① = F\frac{\cos\varrho}{\cos\alpha\cos\varrho - \sin\alpha\sin\varrho} = F\frac{1}{\cos\alpha - \underbrace{\frac{\sin\varrho}{\cos\varrho}}_{\mu}\sin\alpha} = \frac{F}{\cos\alpha - \mu\sin\alpha}$$

$$\cos(\alpha+\varrho) = \cos\alpha\cos\varrho - \sin\alpha\sin\varrho$$

Analog für $M_d = r F \tan(\alpha+\varrho) = r F \frac{\sin\alpha + \mu\cos\alpha}{\cos\alpha - \mu\sin\alpha}$

$$\tan(\alpha+\varrho) = \frac{\tan\varrho + \tan\alpha}{1 - \tan\varrho\tan\alpha}$$

Nun Schraube nach unten drehen:



$$\Sigma F_z = 0 : \quad -F + F_R\cos(\alpha-\varrho) = 0$$

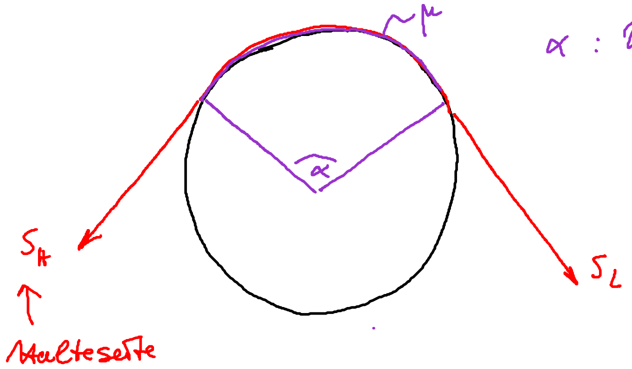
$$\Rightarrow F_R = \frac{F}{\cos(\alpha-\varrho)} \Rightarrow F_N = \frac{\cos\alpha}{\cos(\alpha-\varrho)} F$$

$$F_N = F_R\cos\varrho$$

analog für Moment

$$M_d = r F \tan(\alpha-\varrho)$$

Euler-Eytelweinsche Seilreibung



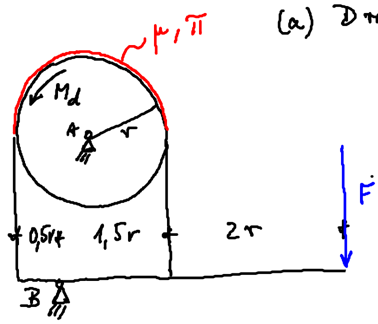
α : Umschlingungswinkel

Antwort:

$$S_L = S_H \exp(\mu \alpha)$$

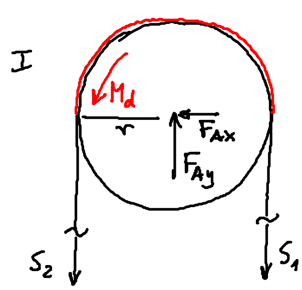
Anwendung der Seilreibungsformel: Seilbremse

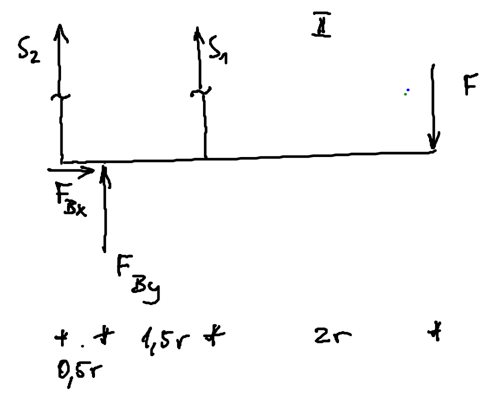
(a) Drehung nach links



Ziel: Berechnung von F um M im Gleichgewicht zu halten

Freischnitt



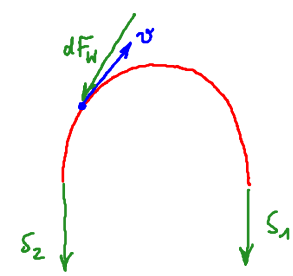


$$\sum M^{(A)} = 0 : \quad M_d + S_2 r - S_1 r = 0 \qquad (1)$$

$$\sum M^{(B)} = 0 : \quad -S_2\, 0{,}5r + S_1\, 1{,}5r - F\, 3{,}5r = 0 \qquad (2)$$

Unbekannte

S_1, S_2, F



Euler – Eytelwein $\quad S_1 = S_2 \exp(\pi \mu) \qquad (3)$

Lastseite $\quad$ Halteseite

$$\mu = \frac{1}{\pi} \ln \frac{1}{3}$$

$\leq 0 \qquad 2$

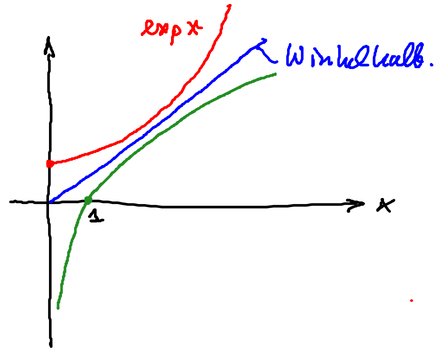
Drehung nach links

$$S_1 = \frac{M_d \exp(\mu\pi)}{r[\exp(\mu\pi) - 1]}, \quad S_2 = \frac{M}{r[\exp(\mu\pi) - 1]}, \quad F = \frac{M}{r} \frac{3\exp(\mu\pi) - 1}{7[\exp(\mu\pi) - 1]}$$

Drehung nach rechts

$$S_1 = \frac{M_d}{r[\exp(\mu\bar{\pi})-1]}, \quad S_2 = \frac{M\exp(\mu\bar{\pi})}{r[\exp(\mu\bar{\pi})-1]}, \quad F = \frac{M}{r}\,\frac{3-\exp(\mu\bar{\pi})}{7[\exp(\mu\bar{\pi})-1]}$$

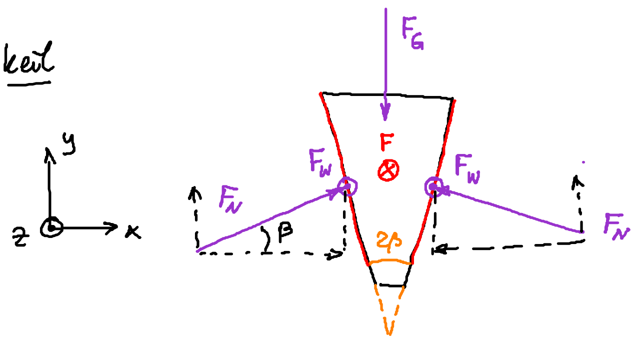
≤ 0



$$3-\exp(\mu\bar{\pi}) = 0$$

$$\mu = \frac{1}{\pi}\ln 3 > 0$$

Keil



$$\Sigma F_z = 0: -F + 2F_W = 0 \quad\Rightarrow\quad F = 2F_W \quad (1)$$

$$\Sigma F_x = 0: \quad F_N\cos\beta - F_N\cos\beta = 0$$

$$\Sigma F_y = 0: \quad -F_G + 2F_N\sin\beta = 0$$

$$\Rightarrow \quad F_N = \frac{F_G}{2\sin\beta} \quad (2)$$

Coulomb $\quad F_W = \mu F_N \quad (3)$

$$\Rightarrow \quad F = 2\mu F_N = \underbrace{\frac{\mu}{\sin\beta}}_{\mu_{eff} > \mu} F_G$$