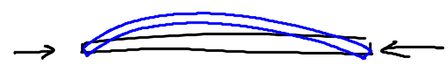
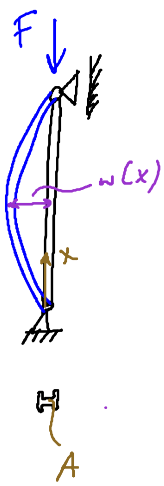
Kinematik & Dynamik

- Knicken
- Euler Hyperbel
- Die 4 Eulerschen Knicktypen

Knicken: Kraft ist senkrecht zur Verschiebung



Freischnitt in dem verformten System
Theorie 2. Ordnung



$M(x) = F w$

$w'' = -\frac{M}{EI}$

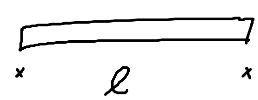
$w'' + \alpha^2 w = 0 \quad , \quad \alpha^2 = \frac{F}{EI}$

Ansatz: $w = A \sin(\alpha x) + B \cos(\alpha x)$ $\quad A, B$: aus den Randbedingungen

$F = n^2 \frac{EI\pi^2}{\ell^2}$ (F_{kr})

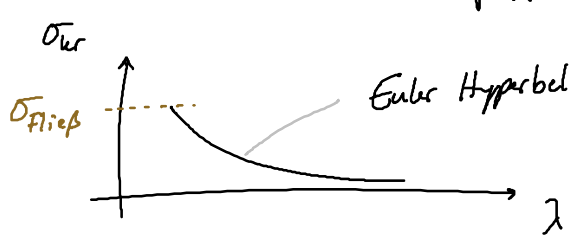
$\sigma_{kr} = \frac{F_{kr}}{A} = \frac{EI\pi^2}{\ell^2 A}$

schlanker Balken

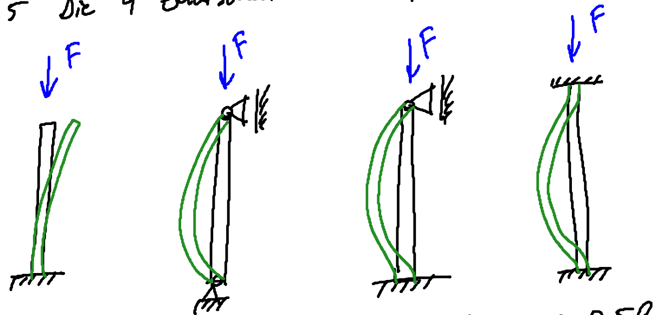


Schlankheitsgrad $\lambda = \frac{\ell}{i}$

$i = \sqrt{\frac{I}{A}}$



2.9.5 Die 4 Eulerschen Knicktypen

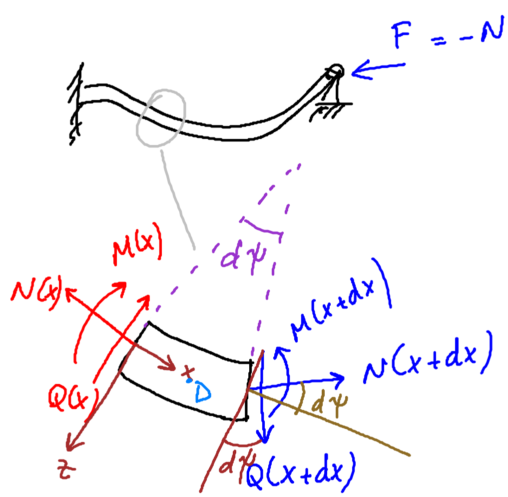


$s = 2\ell$ $\quad$ $s = \ell$ $\quad$ $s \approx 0{,}7\ell$ $\quad$ $s = 0{,}5\ell$

$$F_{kr} = \frac{E I \pi^2}{s^2}$$

s: Knicklänge

Erinnere an Biegung:



$$N(x+dx) = N(x) + \underbrace{\frac{\partial N}{\partial x} dx}_{dN} + \underbrace{\frac{\partial^2 N}{\partial x^2} dx^2 + \ldots}_{\mathcal{O}(dx^2)}$$

$$Q(x+dx) = Q(x) + dQ$$

$$\Sigma F_x = 0 = -N + (N+dN)\underbrace{\cos(d\varphi)}_{=1} + (Q+dQ)\underbrace{\sin(d\varphi)}_{d\varphi}$$

$$Q\,d\varphi + \underbrace{dQ\,d\varphi}_{=0}$$

$$0 = dN + Q\,d\varphi$$

$$-Q\,d\varphi = dN$$

$$\boxed{-Q\frac{d\varphi}{dx} = \frac{dN}{dx}}$$

$$\Sigma M^{(D)} = 0 = -M + M + dM - Q\frac{dx}{2} - (Q+dQ)\frac{dx}{2}$$

$$0 = dM - Q\frac{dx}{2} - Q\frac{dx}{2}$$

$$Q\,dx = dM$$

$$\boxed{Q = \frac{dM}{dx}}$$

$$\Sigma F_z = 0 = -Q + (Q+dQ)\underbrace{\cos(d\varphi)}_{=1} - (N+dN)\underbrace{\sin(d\varphi)}_{d\varphi}$$

$$dN\,d\varphi = \mathcal{O}(d\ldots$$

$$0 = dQ - N\,d\varphi$$

$$\boxed{N\frac{d\varphi}{dx} = \frac{dQ}{dx}}$$

$$w'(x+dx) = w'(x) + \underbrace{dw'}_{-d\varphi}$$

$$\frac{dQ}{dx} = \frac{d^2M}{dx^2} = N\frac{d\varphi}{dx} = -N w''$$

$$M'' = -N w'' = F w''$$
$$M = -w'' \underbrace{EI}_{\text{const.}}$$
$$\Rightarrow \quad -w^{(IV)} EI = F w''$$

$$\boxed{0 = w^{(IV)} + \alpha^2 w''}\ , \quad \alpha^2 = \frac{F}{EI}$$

DGL lösen: $\left(w'' + \alpha^2 w\right)'' = 0$

$w'' + \alpha^2 w = ax + b$ lineare DGL 2. Ordnung, inhomogen!

$w = w_h + w_p$

w_h: homogen, w_p: partikulär

$$(w_h + w_p)'' + \alpha^2 (w_h + w_p) = 0 + ax + b$$

$w_h'' + \alpha^2 w_h = 0$

$w_p'' + \alpha^2 w_p = ax + b$

Ansatz: $w_h = A_1 \cos(\alpha x) + A_2 \sin(\alpha x)$

$c \mathrel{\hat=} \cos(\alpha l)$, $s \mathrel{\hat=} \sin(\alpha l)$

$w_p = A_3 x + A_4$

$w = A_1 \cos(\alpha x) + A_2 \sin(\alpha x) + A_3 x + A_4$

1) $w(x=0) = 0$

2) $w'(x=0) = 0$

3) $w(x=l) = 0$

$M(x=l) = 0 = -w''(x=l) EI$

4) $w''(x=l) = 0$

1) $0 = A_1 \cdot 1 + A_2 \cdot 0 + A_3 \cdot 0 + A_4 \cdot 1$

2) $0 = A_1 \cdot 0 + \alpha A_2 \cdot 1 + A_3 \cdot 1 + A_4 \cdot 0$

3) $0 = A_1 \cdot c + A_2 \cdot s + A_3 l + A_4 \cdot 1$

4) $0 = -A_1 \alpha^2 c - \alpha^2 A_2 s + A_3 \cdot 0 + A_4 \cdot 0$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & \alpha & 1 & 0 \\ c & s & \ell & 1 \\ -\alpha^2 c & -\alpha^2 s & 0 & 0 \end{pmatrix}}_{\det = 0 \;\Rightarrow\; \tan(\alpha\ell) = \alpha\ell \;\Rightarrow\; s = 0{,}7\ell} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix}$$