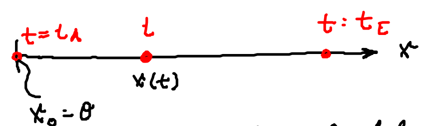
# Vorlesung am 4.5.2018

- Wiederholung Kinematik 1D
- Kinematik 3D
- Kinetik: Newton's Grundgesetze
- Beispiele
- Impulssatz

## Wiederholung 1D eines MP




$x = x(t)$ Ort

$\underbrace{\frac{dx}{dt}}_{\dot{x}(t)} = v(t)$ Geschwindigkeit

$\underbrace{\frac{d^2x}{dt^2}}_{\ddot{x}(t)} = \underbrace{\frac{dv(t)}{dt}}_{\dot{v}(t)} = a(t)$ Beschleunigung

Annahme Beschl. $a$ gegeben; Ziel $v(t)$, $x(t)$

$$\Rightarrow \frac{dv}{dt} = a(t) \Rightarrow dv = a(t)\,dt$$

$$v(t) = \int a(t)\,dt + A$$

$A$ folgt aus sog. AB: $v(t=0) = v_0$

$$\frac{dx}{dt} = v(t) = \int a(t)\,dt + A$$

$$x(t) = \int \left[\int a(t)\,dt\right] dt + At + B$$

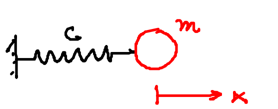
$B$ folgt aus AB: $x(t=0) = x_0$

**Problem:** Manchmal ist $a$ nicht als Zeitfunktion gegeben

(a) $a = a(v(t)) = \frac{dv}{dt}$

$$\Rightarrow \frac{dv}{a(v)} = dt \Rightarrow \int \frac{dv}{a(v)} = t + A$$

(b) $a = a(x(t))$




Kettenregel

$$a(t) = -\omega^2 x(t) = \frac{dv}{dt} = \frac{dv(x)}{dx}\underbrace{\frac{dx(t)}{dt}}_{v} \Rightarrow \frac{dv}{dx}v = -\omega^2 x \quad \Big| \int$$

$$v\,dv = -\omega^2 x\,dx$$

$$\frac{v^2}{2} = -\omega^2 \frac{x^2}{2} + C$$

Annahme zur Best. von Konstante $C$:

$$\left.\begin{array}{l} v_0 = v(t=0) = 0 \\ x(t=0) = x_0 \end{array}\right\}$$

$$v^2(t) = -\omega^2 x^2(t) + C$$

$$t=0: \quad 0 = -\omega^2 x_0^2 + C \Rightarrow C = \omega^2 x_0^2 \quad \Big\} \quad v(t) = \pm\omega\sqrt{x_0^2 - x^2(t)}$$

$$v(t) = \frac{dx}{dt} = \omega\sqrt{x_0^2 - x^2(t)} \Rightarrow \frac{dx}{\sqrt{x_0^2 - x^2(t)}} = \omega\,dt \quad \Big| \int_{\tilde{x} = x(0) = x_0}^{\tilde{x} = x(t)} \frac{d\tilde{x}}{\sqrt{x_0^2 - \tilde{x}^2(t)}} = \omega \int_{\tilde{t}=0}^{\tilde{t}=t} d\tilde{t}$$

$$\arcsin \frac{\tilde{x}}{x_0}\Bigg|_{\tilde{x} = x_0}^{\tilde{x} = x(t)}$$

$$\arcsin \frac{x(t)}{x_0} - \arcsin \underbrace{\frac{x_0}{x_0}}_{1} = \omega \tilde{t}\,\Big|_{\tilde{t}=0}^{\tilde{t}=t} = \omega t$$

$$\underbrace{\underbrace{\qquad}_{\frac{\pi}{2}}\qquad}_{\arccos \frac{x(t)}{x_0}}$$

$$\Rightarrow \quad \frac{x(t)}{x_0} = \cos(\omega t)$$

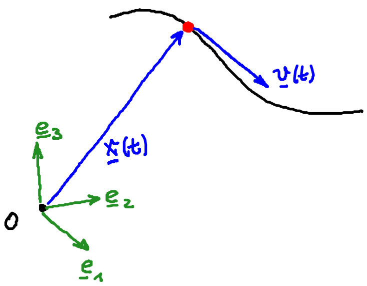
$$x(t) = x_0 \cos(\omega t)$$

$$v(t) = \frac{dx(t)}{dt} = -x_0 \omega \sin(\omega t)$$

$$v(t) = \omega \sqrt{x_0^2 - x_0^2 \cos^2(\omega t)} =$$

$$= \omega x_0 \underbrace{\sqrt{1-\cos^2 \omega t}}_{\sin \omega t}$$

## Position, Geschwindigkeit, Beschleunigung in 3D




Ortsvektor $\quad \underline{x} = \underline{x}(t)$

Geschwindigkeitsvektor $\quad \frac{d\underline{x}(t)}{dt} = \underline{v}(t)$

Beschleunigungsvektor $\quad \frac{d^2\underline{x}(t)}{dt^2} = \frac{d\underline{v}(t)}{dt} = \underline{a}(t)$

Berechnung: $\quad \underline{x}(t) = x_1(t)\,\underline{e}_1 + x_2(t)\,\underline{e}_2 + x_3(t)\,\underline{e}_3$

$$\underline{F} = (F_1, F_2, F_3) = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

$$= F_1 \underline{e}_1 + F_2 \underline{e}_2 + F_3 \underline{e}_3$$

$$\underline{v}(t) = \frac{d\underline{x}(t)}{dt} = \frac{d}{dt}(\quad) = \underbrace{\frac{dx_1}{dt}}_{v_1(t)} \underline{e}_1 + \underbrace{\frac{dx_2}{dt}}_{v_2(t)} \underline{e}_2 + \underbrace{\frac{dx_3}{dt}}_{v_3(t)} \underline{e}_3$$

$$= \frac{dx_i(t)}{dt} \underline{e}_i = v_i(t)\, \underline{e}_i$$

$$\underline{a}(t) = \frac{d\underline{v}(t)}{dt} = \underbrace{\frac{dv_1}{dt}}_{a_1(t)} \underline{e}_1 + \underbrace{\frac{dv_2}{dt}}_{a_2(t)} \underline{e}_2 + \underbrace{\frac{dv_3}{dt}}_{a_3(t)} \underline{e}_3 = \frac{dv_i}{dt} \underline{e}_i$$

$$= \frac{d^2 x_1}{dt^2} \underline{e}_1 + \frac{d^2 x_2}{dt^2} \underline{e}_2 + \frac{d^2 x_3}{dt^2} \underline{e}_3 = \frac{d^2 x_i}{dt^2} \underline{e}_i$$

$$v = |\underline{v}| = \sqrt{\underline{v} \cdot \underline{v}} = \sqrt{v_1 v_1 + v_2 v_2 + v_3 v_3}$$

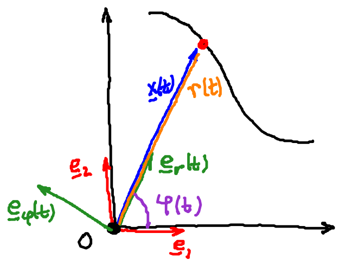
$$= \sqrt{v_i v_i}$$

$$\underline{v} = v_i \underline{e}_i \quad , \quad \underline{v} \cdot \underline{v} = v_i \underline{e}_i \cdot v_j \underline{e}_j =$$

$$= v_i v_j \underbrace{\underline{e}_i \cdot \underline{e}_j}_{\delta_{ij} = \begin{cases} 1 & \text{für } i = j \\ 0 & \text{sonst} \end{cases}} = v_i v_i$$

## Weitere Koordinatensysteme

## (a) Ebene Polarkoordinaten




$$\underline{x}(t) = r(t)\,\underline{e}_r(t) = (r(t), 0) = r(t)\,\underline{e}_r(t) + 0\,\underline{e}_\varphi(t)$$

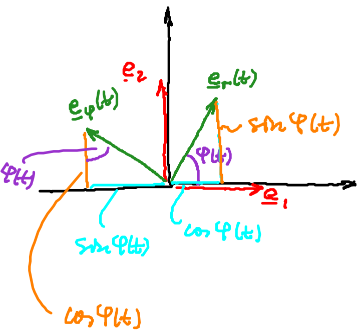
$$\underline{v}(t) = \frac{d\underline{x}(t)}{dt} = \frac{d}{dt}\left[r(t)\,\underline{e}_r(t)\right] = \frac{dr(t)}{dt}\,\underline{e}_r(t) + r(t)\,\frac{d\underline{e}_r(t)}{dt}$$

$$= \dot{r}(t)\,\underline{e}_r(t) + r(t)\,\omega(t)\,\underline{e}_\varphi(t) = (\dot{r}(t), r(t)\,\omega(t))$$

$$\underline{a}(t) = \ddot{r}\,\underline{e}_r(t) + \dot{r}\,\frac{d\underline{e}_r}{dt} + \dot{r}(t)\,\omega(t)\,\underline{e}_\varphi(t) + r\,\dot{\omega}\,\underline{e}_\varphi + r\,\omega\,\frac{d\underline{e}_\varphi}{dt}$$

$$= \underbrace{\ddot{r}\,\underline{e}_r}_{\text{Radial-}} + \underbrace{2\dot{r}\,\omega\,\underline{e}_\varphi}_{\text{Coriolis-}} + \underbrace{r\,\dot{\omega}\,\underline{e}_\varphi}_{\text{Euler-}} \underbrace{- r\,\omega^2\,\underline{e}_r}_{\text{Zentripetal}} = (\ddot{r} - r\omega^2, 2\dot{r}\omega + r\dot{\omega})$$

beschleunigungs




$$\underline{e}_r(t) = \cos\varphi(t)\,\underline{e}_1 + \sin\varphi(t)\,\underline{e}_2$$

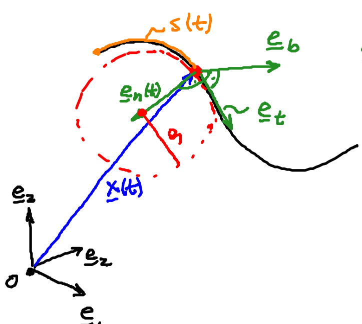
$$\frac{d\underline{e}_r(t)}{dt} = -\overbrace{\frac{d\varphi}{dt}}^{\omega(t)}\sin\varphi(t)\,\underline{e}_1 + \frac{d\varphi}{dt}\cos\varphi(t)\,\underline{e}_2$$

$$= \omega(t)\left[-\sin\varphi(t)\,\underline{e}_1 + \cos\varphi(t)\,\underline{e}_2\right] = \omega(t)\,\underline{e}_\varphi(t)$$

$$\underline{e}_\varphi(t) = -\sin\varphi(t)\,\underline{e}_1 + \cos\varphi(t)\,\underline{e}_2$$

$$\frac{d\underline{e}_\varphi(t)}{dt} = -\omega(t)\left[\cos\varphi(t)\,\underline{e}_1 + \sin\varphi(t)\,\underline{e}_2\right] = -\omega(t)\,\underline{e}_r(t)$$

## Eigenkoordinaten, natürliche Koordinaten




$\underline{e}_b = \underline{e}_t \times \underline{e}_n$

Binormalenvektor

$$\underline{x} = \underline{x}(s)$$

$$\underline{v} = v(t)\,\underline{e}_t(t) = \dot{s}(t)\,\underline{e}_t(t)$$

$$\underline{a} = \frac{d\underline{v}}{dt} = \frac{d}{dt}\left[\dot{s}(t)\,\underline{e}_t(t)\right]$$

$$= \ddot{s}(t)\,\underline{e}_t(t) + \dot{s}\,\underbrace{\frac{d\underline{e}_t}{dt}}_{\frac{\dot{s}}{\varrho}\,\underline{e}_n}$$

$$= \ddot{s}(t)\,\underline{e}_t(t) + \underbrace{\frac{\dot{s}^2}{\varrho}}_{\frac{v^2}{\varrho}}\,\underline{e}_n(t)$$

$$\dot{s} = v$$

## 3.1.2 Kinetik des MP

Impuls : $\underline{p} = m\,\underline{v} \quad \Rightarrow \quad \frac{d}{dt}\,\underline{p} = \frac{d}{dt}(m\,\underline{v}(t)) = m\,\frac{d\underline{v}(t)}{dt} = m\,\underline{a}(t)$

$$\Downarrow$$

Lex II : $\frac{d\underline{p}}{dt} = \underline{F} \quad \Rightarrow \quad m\,\underline{a}(t) = \underline{F}(t)$

$$\Downarrow \quad \int_{\tilde{t}=0}^{\tilde{t}=t} d\tilde{t}$$

$$\underline{p}(t) - \underline{p}(0) = \underbrace{\int_{\tilde{t}=0}^{\tilde{t}=t} \underline{F}(\tilde{t})\,d\tilde{t}}_{\text{Kraftstoß}}$$

$$\underline{F}(t) = 0 \quad \Rightarrow \quad \underline{p}(t) = \underline{p}(0)$$

$$\Downarrow$$

Impulssatz $\quad \cancel{m}\,\underline{v}(t) = \cancel{m}\,\underline{v}(0)$