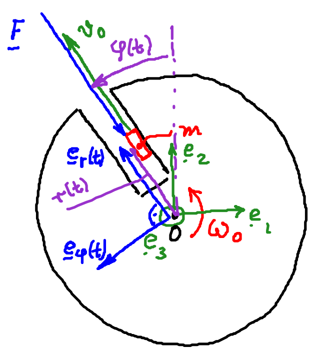
Vorlesung am 11.5.2018

- Wiederholung: Bewegungsgleichung für MP aufstellen und lösen
- Impulssatz und Stoße
- Energie- und Arbeitssatz

Wiederholung

1. Geführte Bewegung

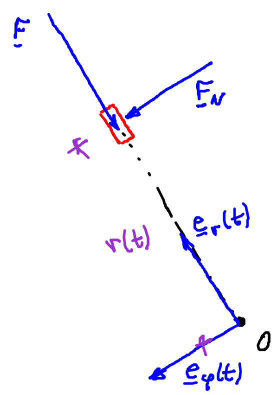


Vorgegeben: ω_0

Zielgröße: $v_0 = \text{const.}$

Gesucht: $\underline{F}(t)$

1. Freischnitt



2. Gleichungen

Kinematik Polarkoordinaten

$$\underline{x}(t) = r(t)\,\underline{e}_r(t)$$

$$\underline{v}(t) = \frac{d\underline{x}}{dt} = \dot{r}(t)\,\underline{e}_r(t) + r(t)\,\omega(t)\,\underline{e}_\varphi(t)$$

$$\frac{d\underline{e}_r}{dt} = \dot{\varphi}\,\underline{e}_\varphi = \omega(t)\,\underline{e}_\varphi(t)$$

$$\underline{a}(t) = \frac{d\underline{v}}{dt} = \ddot{r}\,\underline{e}_r + 2\dot{r}\,\omega\,\underline{e}_\varphi + r\dot{\omega}\,\underline{e}_\varphi - r\omega^2\underline{e}_r$$

$$\frac{d\underline{e}_\varphi}{dt} = -\omega\,\underline{e}_r$$

Speziell:

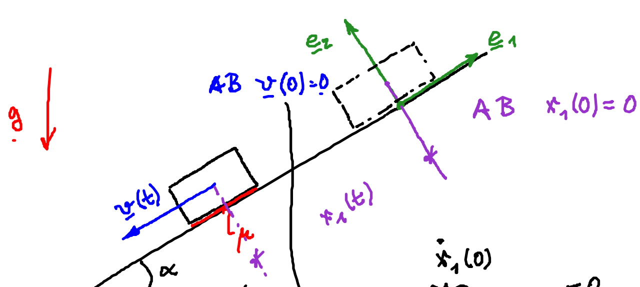
$$\underline{a}(t) = \underbrace{\ddot{r}}_{\frac{d}{dt}\dot{r} = \frac{d}{dt}v_0}\underline{e}_r + 2\underbrace{\dot{r}}_{v_0}\omega_0\underline{e}_\varphi - r\omega_0^2\underline{e}_r = 2v_0\omega_0\underline{e}_\varphi - r\omega_0^2\underline{e}_r$$

Newton: $m\underline{a} = \Sigma\underline{F}$: $2m v_0 \omega_0 \underline{e}_\varphi - m r \omega_0^2 \underline{e}_r = -F(t)\,\underline{e}_r + F_N\,\underline{e}_\varphi$

$\underline{e}_\varphi$: $2 m v_0 \omega_0 = F_N \Rightarrow F_N = 2 m v_0 \omega_0$

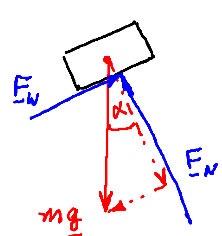
$\underline{e}_r$: $-m r \omega_0^2 = -F(t) \Rightarrow F(t) = m\,r(t)\,\omega_0^2$

2. Geführte Bewegung mit Coulombreibung



$\underline{v}(0) = v_1(0)\,\underline{e}_1 + \cancel{v_2(0)}^{=0}\,\underline{e}_2 = \underline{0}$

1. Freischnitt



2. Gleichungen

Kinematik

$$\underline{x} = x_1(t)\,\underline{e}_1 + x_2(t)\,\underline{e}_2$$

Kinematischer Zwang

$$x_2(t) = 0$$

Speziell: $\underline{v} = \dot{x}_1(t)\,\underline{e}_1$

$$\underline{a} = \ddot{x}_1(t)\,\underline{e}_1$$

$m\underline{a} = \sum \underline{F}$: $\quad m\ddot{x}_1(t)\,\underline{e}_1 = -mg\sin\alpha\,\underline{e}_1 + F_W\,\underline{e}_1 - mg\cos\alpha\,\underline{e}_2 + F_N\,\underline{e}_2$

Coulomb:

$F_W = \mu F_N \quad (3)$

$\underline{e}_1$: $\quad m\ddot{x}_1(t) = -mg\sin\alpha + F_W \quad (1)$

$\underline{e}_2$: $\quad 0 = -mg\cos\alpha + F_N \Rightarrow F_N = mg\cos\alpha \quad (2)$

(1) bis (3): $\quad \cancel{m}\ddot{x}_1(t) = -mg\sin\alpha + \mu mg\cos\alpha = -\cancel{m}g[\sin\alpha - \mu\cos\alpha]$

$$\ddot{x}_1(t) = -g[\sin\alpha - \mu\cos\alpha]$$

$$\dot{x}_1(t) = -g[\sin\alpha - \mu\cos\alpha]\,t + A_1 \Rightarrow A_1 = 0$$

$\dot{x}_1(0) = 0$

$$\frac{dx_1}{dt} = \dot{x}_1(t) = -g[\sin\alpha - \mu\cos\alpha]\,t$$

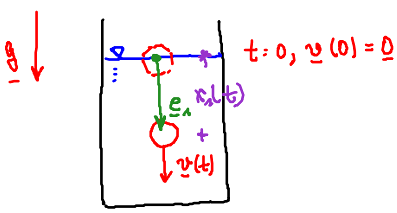
$$\Rightarrow x_1(t) = -\frac{g}{2}[\sin\alpha - \mu\cos\alpha]\,t^2 + B_1 \Rightarrow B_1 = 0$$

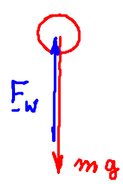
$x_1(0) = 0$

$$\Rightarrow x_1(t) = -\frac{g}{2}[\sin\alpha - \mu\cos\alpha]\,t^2$$

3. Stokessche Reibung

1. Freischnitt





Stokes: $\quad F_W(t) = k\,\underbrace{v_1(t)}_{\dot{x}_1(t)}$

2. Gleichungen

Kinematik

$\underline{x}(t) = x_1(t)\,\underline{e}_1$

$\underline{v}(t) = \dot{x}_1(t)\,\underline{e}_1$

$\underline{a}(t) = \ddot{x}_1(t)\,\underline{e}_1$

Newton:

$m\underline{a} = \sum\underline{F} \Rightarrow m\ddot{x}_1(t)\,\underline{e}_1 = mg\,\underline{e}_1 - F_W(t)\,\underline{e}_1$

$\underline{e}_1$: $\quad \Rightarrow m\ddot{x}_1(t) = mg - k\dot{x}_1 \quad \Rightarrow \quad \frac{dv_1(t)}{dt} = g - \frac{k}{m}v_1(t)$

$\Downarrow$

Substituiere: $\quad \tilde{v}_1(t) = v_1(t) - g\frac{m}{k} \quad \#$

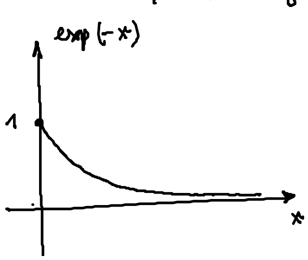
$$\frac{d\tilde{v}_1(t)}{dt} = \frac{dv_1(t)}{dt} \quad \Rightarrow \quad \frac{d\tilde{v}_1(t)}{dt} = g - \frac{k}{m}\left[\tilde{v}_1(t) + g\frac{m}{k}\right] =$$

$\tilde{v}_1(0) = \underbrace{v_1(0)}_{=0} - g\frac{m}{k}$

$$= \cancel{g} - \frac{k}{m}\tilde{v}_1(t) - \cancel{g} = -\frac{k}{m}\tilde{v}_1(t)$$

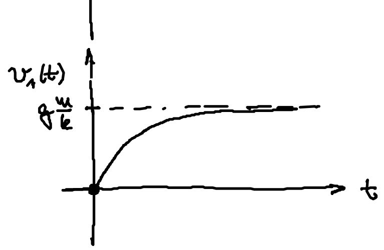
$\tilde{t} = t$

$\Rightarrow$ Trennung der Variablen $\quad \frac{d\tilde{v}_1(t)}{\tilde{v}_1(t)} = -\frac{k}{m}\,dt \quad \Big| \int_{\tilde{t}=0}^{\tilde{t}=t} \text{—"—}\, d\tilde{t}$



$$\ln\tilde{v}_1(\tilde{t})\Big|_{\tilde{t}=0}^{\tilde{t}=t} = -\frac{k}{m}\tilde{t}\Big|_{\tilde{t}=0}^{\tilde{t}=t}$$

$$\ln\frac{\tilde{v}_1(t)}{\tilde{v}_1(0)} = -\frac{k}{m}t \quad \Rightarrow \quad \tilde{v}_1(t) = \tilde{v}_1(0)\exp\left[-\frac{k}{m}t\right]$$



$$\# \Rightarrow v_1(t) = g\frac{m}{k}\left[1 - \exp\left[-\frac{k}{m}t\right]\right], \quad v_1(t \gg 0) = g\frac{m}{k}$$

$$\frac{dx_1(t)}{dt} = g\frac{m}{k}\left[1 - \exp\left[-\frac{k}{m}t\right]\right] \qquad \int_{\tilde{t}=0}^{\tilde{t}=t} -\|- \, d\tilde{t}$$

$$\Rightarrow x_1(t) - \underbrace{\cancel{x_1(0)}}_{=0} = g\frac{m}{k}\left[t + \frac{m}{k}\left(\exp\left(-\frac{k}{m}t\right) - 1\right)\right]$$

$$x_1(t \gg 0) = \mathcal{O}(t)$$

3.1.3 Der Impulssatz und seine Anwendung auf Stöße

Impulssatz: Impuls $\underline{p} := m\underline{v}$ ← konstant

Newton $\frac{d\underline{p}}{dt} = \underline{F} \Rightarrow m\overbrace{\frac{d\underline{v}}{dt}}^{\underline{a}} = \underline{F}(t)$

$\underline{p} = m\underline{v}(t)$

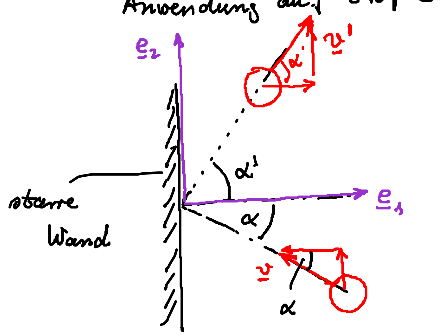
$$\Rightarrow \frac{d}{dt}(m\underline{v}) = \underline{F} \qquad \Big| \int -\|- dt$$

$$\Rightarrow m\underline{v}\Big|_{\tilde{t}=0}^{\tilde{t}=t} = \int_{\tilde{t}=0}^{\tilde{t}=t} \underline{F}(\tilde{t})\,d\tilde{t} \Rightarrow \overbrace{m\underline{v}(t)}^{\underline{p}(t)} = \overbrace{m\underline{v}(0)}^{\underline{p}(0)} + \int_{\tilde{t}=0}^{\tilde{t}=t} \underline{F}(\tilde{t})\,d\tilde{t}$$

Das ist der Impulssatz

Def.: Kraftstoß $\underline{K} := \int_{\tilde{t}=0}^{\tilde{t}=t} \underline{F}(\tilde{t})\,d\tilde{t} \Rightarrow \underline{p}(t) = \underline{p}(0) + \underline{K}$

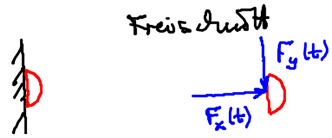
Anwendung auf Stöße



Ziel: Berechnung von $\underline{v}'$ (Geschwindigkeit nach Betrag und Richtung, d.h. v', α') durch $\underline{v}$ vor dem Stoß (also nach Betrag und Richtung, d.h. v, α)

Entwicklung der Kraft über der Zeit

Freischnitt



Annahme: Die Wand sei erst einmal glatt, d.h. $F_y(t) = 0$

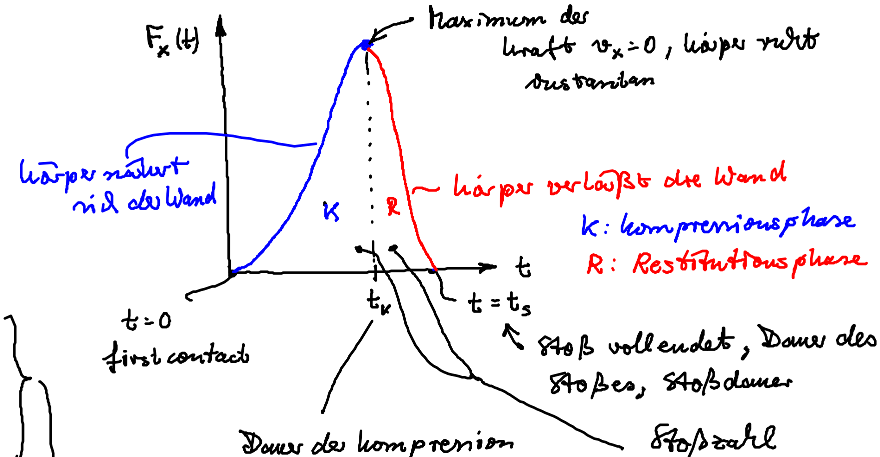
$\Rightarrow K_y(t) = 0$

Impulssatz

$$v_y' = v_y$$

$$m v_x' = -m v_x + \int_{\tilde t = 0}^{\tilde t = t} F_x(\tilde t)\, d\tilde t$$

In den beiden Abschnitten K und R wird



$$K: \quad m\,0 = -m v_x + K_x^K \quad , \quad K_x^K = \int_{\tilde t = 0}^{\tilde t = t_K} F_x(\tilde t)\, d\tilde t$$

$$R: \quad m v_x' = m \cdot 0 + K_x^R \quad , \quad K_x^R = \int_{\tilde t = t_K}^{\tilde t = t_S} F_x(\tilde t)\, d\tilde t$$

$$\Rightarrow \quad m v_x = K_x^K \qquad m v_x' = K_x^R$$

$$0 \le e = \frac{K_x^R}{K_x^K} \le 1$$

voll plast. voll elast.

Winkel:

$$\left.\begin{array}{l} v_x = v \cos\alpha \\ v_x' = v' \cos\alpha' \end{array}\right\} \quad v' \cos\alpha' = e\, v \cos\alpha \quad (1)$$

$$\left.\begin{array}{l} v_y = v \sin\alpha \\ v_y' = v' \sin\alpha' \end{array}\right\} \quad \text{mit } v_y = v_y' \Rightarrow \quad v \sin\alpha = v' \sin\alpha' \; (2) \text{ glatte Wand}$$

$$(2)/(1) \quad \Rightarrow \quad \tan\alpha' = \frac{1}{e} \tan\alpha$$