Vorlesung am 25.5.2018

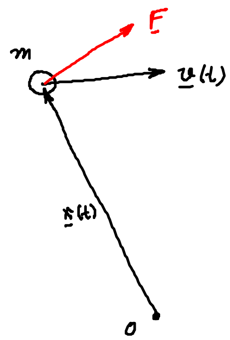
- Wiederholung Drehimpuls
- Massenpunktsysteme Kinematik und Kinetik

Wiederholung

Newton: $m\frac{d\underline{v}}{dt} = \underline{F}$

Impuls $\underline{p}(t) := m\,\underline{v}(t)$

Drehimpuls / Drallsatz



Drehimpuls $\underline{L}^{(0)} := \underline{x}(t)\times\underline{p}(t) \Rightarrow \frac{d\underline{L}^{(0)}}{dt} = \frac{d}{dt}\left[m\,\underline{x}(t)\times\underline{v}(t)\right] =$

$$= \left[\underline{x}(t)\times\left[m\frac{d\underline{v}}{dt}\right]\right] =$$

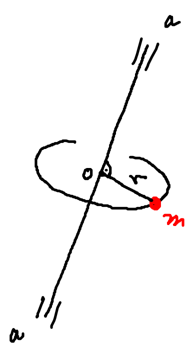
$$= \underline{x}(t)\times\underline{F}(t) = \underline{M}^{(0)}$$

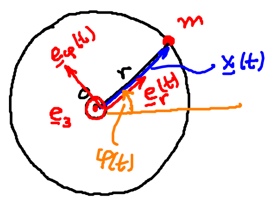
$$\frac{d\underline{L}^{(0)}}{dt} = \underline{M}^{(0)}$$

Drallsatz

$$\Rightarrow \underline{L}^{(0)}(t) - \underline{L}^{(0)}(0) = \int_{\tilde{t}=0}^{\tilde{t}=t}\underline{M}^{(0)}(\tilde{t})\,d\tilde{t} \underset{\text{ohne Momente}}{=} \underline{0} \Rightarrow \underline{L}^{(0)}(t) = \underline{L}^{(0)}(0)$$

$\underline{x}(t)$





$$\underline{L}^{(0)} = \underline{x}(t)\times\underline{p}(t) = m\,\underline{x}(t)\times\underline{v}(t)$$

$$= m\,r^2\omega(t)\,\underbrace{\underline{e}_r(t)\times\underline{e}_\varphi(t)}_{\underline{e}_3} = m\,r^2\omega(t)\,\underline{e}_3$$

$$\underline{x}(t) = r\,\underline{e}_r(t)$$

$$\underline{v}(t) = \dot{\underline{x}}(t) = r\,\omega(t)\,\underline{e}_\varphi(t)$$

$$\frac{d\underline{e}_r}{dt} = \dot{\varphi}(t)\,\underline{e}_\varphi(t) = \omega(t)\,\underline{e}_\varphi(t)$$

$$\underline{L}^{(0)}(0) = m\,r_0^2\underbrace{\omega(0)}_{\omega_0}\underline{e}_3 \overset{!}{=} \underline{L}^{(0)}(t_1) = m\,r_1^2\underbrace{\omega(t_1)}_{\omega_1}\underline{e}_3$$

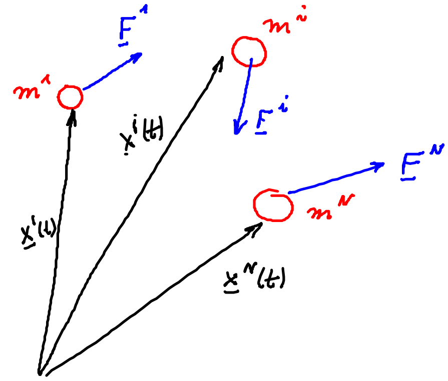
$$\Rightarrow r_0^2\omega_0 = r_1^2\omega_1$$

$$\Rightarrow \omega_1 = \left(\frac{r_0}{r_1}\right)^2\omega_0$$

3.2.2 Kinetik

Newton

$$m^i \ddot{\underline{x}}^i = \underline{F}^i + \sum_{\substack{j=1 \\ i \neq j}}^{N} \underline{F}^{ij} \quad , \quad i = 1, \dots, N$$

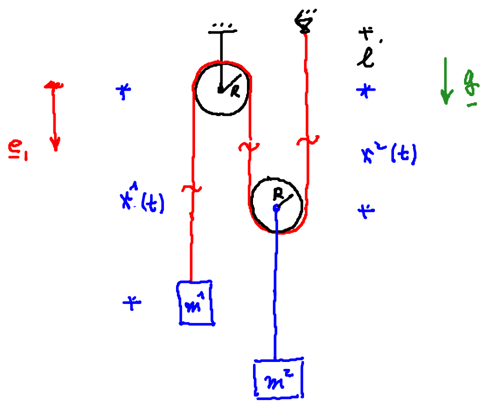


$$\underline{F}^{ij} = - \underline{F}^{ji}$$

Kraft vom MP j auf MP i ausgeübt

Kraft vom MP i auf MP j

Beispiel Umlenkrollen



1. Nebenbedingung

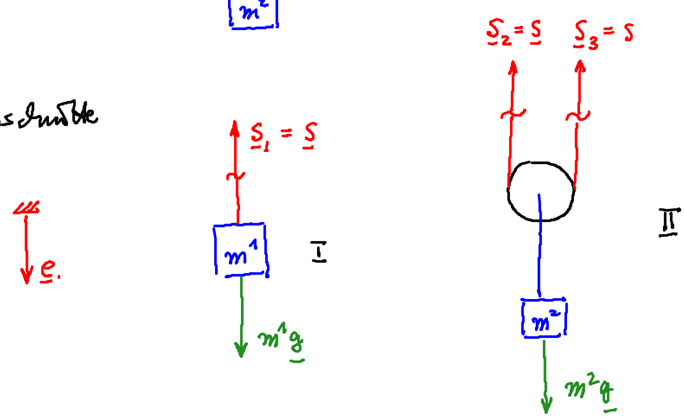
$$\ell = x^1(t) + 2x^2(t) + 2\pi R + \ell' \quad \Big| \frac{d}{dt}$$

$$\Rightarrow 0 = \dot{x}^1(t) + 2\dot{x}^2(t) \Rightarrow \dot{x}^2 = -\frac{1}{2}\dot{x}^1 \quad (3) \qquad \dot{x}^2 = \frac{dx^2}{dt}$$

$$\Rightarrow dx^2 = -\frac{1}{2} dx^1$$

2. Nebenbedingung
Rollbedingung $\underline{S}_1 = \underline{S}_2 = \underline{S}_3 = \underline{S}$

Freischnitt



$$\text{I}: \; e_1: \quad m^1 \ddot{x}^1 = -S + m^1 g \qquad (1)$$

$$\text{II}: \; e_1: \quad m^2 \ddot{x}^2 = -2S + m^2 g \qquad (2)$$

Unbekannte: x^1, x^2, S

$$(2) \Rightarrow S = -\frac{m^2}{2}\ddot{x}^2 + \frac{m^2}{2} g = \frac{m^2}{4}\ddot{x}^1 + \frac{m^2}{2} g \quad \text{in (1)}: \quad m^1 \ddot{x}^1 = -\frac{m^2}{4}\ddot{x}^1 - \frac{m^2}{2} g + m^1 g$$

$$\frac{4m^1 + m^2}{4}\ddot{x}^1 = \frac{2m^1 - m^2}{2} g$$

$$\Rightarrow \ddot{x}^1 = \frac{2(2m^1 - m^2)}{4m^1 + m^2} g \qquad \ddot{x}^2 = -\frac{1}{2}\ddot{x}^1 = -\frac{2m^1 - m^2}{4m^1 + m^2} g$$

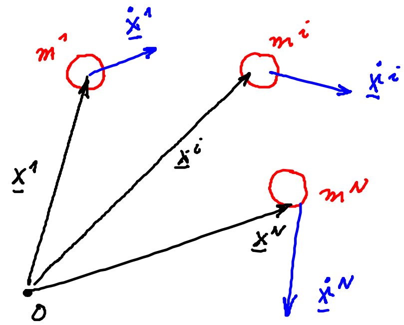
3.2.3 Impuls- und SP-Satz für MP-Systeme

$$i = 1,\dots,N \qquad m^i \ddot{\underline{x}}^i = \underline{F}^i + \sum_j \underline{F}^{ij} \qquad \Big| \sum_i$$

$$\sum_i m^i \ddot{\underline{x}}^i = \underbrace{\sum_i \underline{F}^i}_{\underline{F}} + \underbrace{\sum\sum_{i,j} \underline{F}^{ij}}_{\text{WW-Prinzip} = \underline{0}} \qquad m = \sum_i m^i$$

$$\left.\begin{array}{l} m \underbrace{\dfrac{\sum_i m^i \ddot{\underline{x}}^i}{\sum_i m^i}}_{\frac{d^2}{dt^2}\left[\underbrace{\frac{\sum_i m^i \underline{x}^i}{\sum_i m^i}}_{\underline{x}^s}\right]} = \underline{F} \end{array}\right\} \Rightarrow \quad m \ddot{\underline{x}}^s = \underline{F}$$

3.2.4 Drehimpulssatz für MP-Systeme

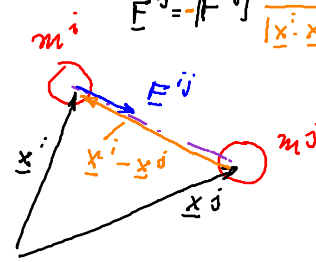


$$\underline{p}^i = m^i \dot{\underline{x}}^i$$

$$\underline{L}^{(0)} = \sum_i \underline{x}^i \times \underline{p}^i = \sum_i m^i \underline{x}^i \times \dot{\underline{x}}^i$$

$$\Rightarrow \frac{d\underline{L}^{(0)}}{dt} = \sum_i m^i \Big(\underbrace{\dot{\underline{x}}^i \times \dot{\underline{x}}^i}_{=\underline{0}} + \underline{x}^i \times \ddot{\underline{x}}^i \Big) =$$

$$\underline{F}^{ij} = -|\underline{F}^{ij}| \frac{\underline{x}^i - \underline{x}^j}{|\underline{x}^i - \underline{x}^j|}$$



$$= \sum_i \underline{x}^i \times \Big[\underline{F}^i + \sum_j \underline{F}^{ij} \Big] = \overbrace{\sum_i \underbrace{\underline{x}^i \times \underline{F}^i}_{\underline{M}^{(0),i}}}^{\underline{M}^{(0)}} + \sum_{i,j} \underline{x}^i \times \underline{F}^{ij}$$

Doppelsumme: $\underline{x}^i \times \underline{F}^{ij} + \underline{x}^j \times \underline{F}^{ji} = (\underline{x}^i - \underline{x}^j) \times \underline{F}^{ij} = \underline{0}$

$\underline{F}^{ji} = -\underline{F}^{ij}$

Das verschwindet für Zentralwechselwirkung

$$\underline{F}^{ij} = -|\underline{F}^{ij}| \frac{\underline{x}^i - \underline{x}^j}{|\underline{x}^i - \underline{x}^j|}$$

$$\Rightarrow \quad \frac{d\underline{L}^{(0)}}{dt} = \underline{M}^{(0)}$$