Vorlesung 1.6.2018

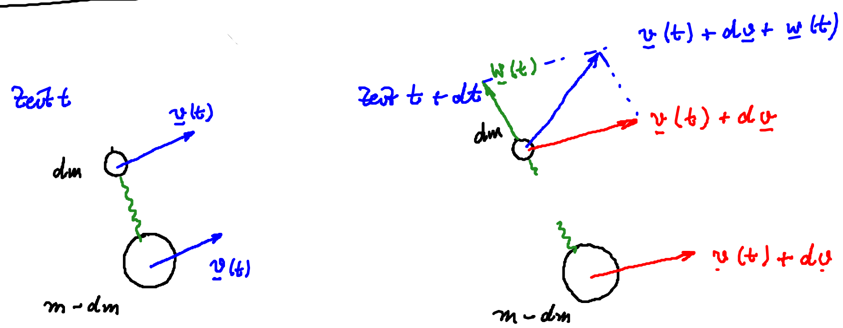
- Anwendung zum Impulssatz: Systeme mit veränderlicher Masse
- Kinematik des starren Körpers

3.2.7 Körper mit zeitveränderlicher Masse

Impulssatz

$$\frac{d\underline{p}(t)}{dt} = \underline{F}(t) \qquad \Big| \int_t^{t+dt}$$

$$\underline{p}^{tot}(t+dt) - \underline{p}^{tot}(t) = \underline{F}\,dt$$



$$\underline{p}^{tot}(t+dt) - \underline{p}^{tot}(t) = (m-dm)(\underline{v}(t)+d\underline{v}) + dm\,(\underline{v}(t) + d\underline{v} + \underline{w}(t)) - (m-dm)\,\underline{v}(t) - dm\,\underline{v}(t) =$$

$$= \cancel{m\underline{v}(t)}^{1} - \cancel{dm\,\underline{v}}^{2} + m\,d\underline{v} - \cancel{dm\,d\underline{v}}^{3} + \cancel{dm\,\underline{v}}^{2} + \cancel{dm\,d\underline{v}}^{3} + dm\,\underline{w} - \cancel{m\underline{v}}^{1} + \cancel{dm\,\underline{v}}^{4} - \cancel{dm\,\underline{v}}^{4} =$$

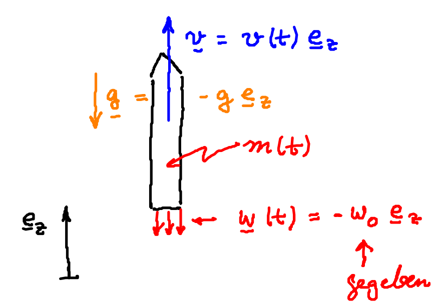
$$= \boxed{m\,d\underline{v} + dm\,\underline{w} = \underline{F}\,dt}$$

$$(1) \quad m(t)\frac{d\underline{v}(t)}{dt} = \underline{F}(t) - \underline{w}(t)\frac{dm}{dt} \quad = \underline{S} \ \text{(Schub)}$$

Lösung für einen konstanten Massenausstoß

$$\frac{dm}{dt} = \mu(t) = -\mu_0 \quad \Rightarrow \quad m(t) - m(0) = -\mu_0 t$$

$$\Rightarrow \quad m(t) = m(0) - \mu_0 t$$



$$(1) \Rightarrow \quad [m(0) - \mu_0 t]\frac{dv(t)}{dt}\underline{e}_z = -[m(0) - \mu_0 t]\,g\,\underline{e}_z + w_0\mu_0\,\underline{e}_z$$

$$\Rightarrow \quad \frac{dv(t)}{dt} = -g + \frac{w_0\mu_0}{m(0) - \mu_0 t} \quad \Rightarrow \quad \boxed{v(t) = -gt + w_0 \ln\frac{m(0)}{m(0) - \mu_0 t}}$$

$$-\frac{1}{\mu_0}\ln|m(0) - \mu_0\tilde{t}|\Big|_{\tilde{t}=0}^{\tilde{t}=t} \qquad \int_{\tilde{t}=0}^{\tilde{t}=t} \text{-"-}\,d\tilde{t}$$

Def.: Brennzeit t_E def. durch

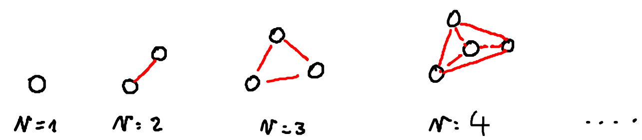
$$m(t_E) = m(0) - \mu_0 t_E = m_E$$

$$\Rightarrow \quad v(t_E) = w_0 \ln\frac{m(0)}{m_E} - g\,\frac{m(0) - m_E}{\mu_0}$$

3.3 Dynamik des starren Körpers

3.3.1 Starrkörperkinematik

Def. starrer Körper

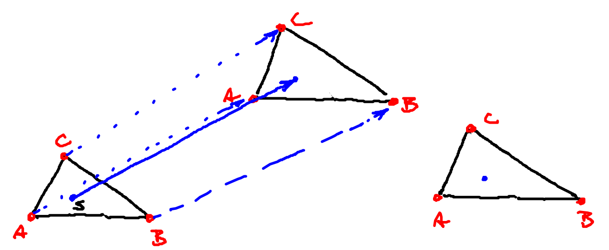


FHG	$f=3$	$f=3+2$	$f=3+3$	$f=3+3$
	3 trans	3 trans + 2 rot	3 trans + 3 rot	3 trans + 3 rot → FHG des starren Körpers

geometrische Untersuchung der Bewegung eines starren Körper

(a) reine Translationsbewegung

reine Verschiebung gleich für alle Pkt. des starren Körpers

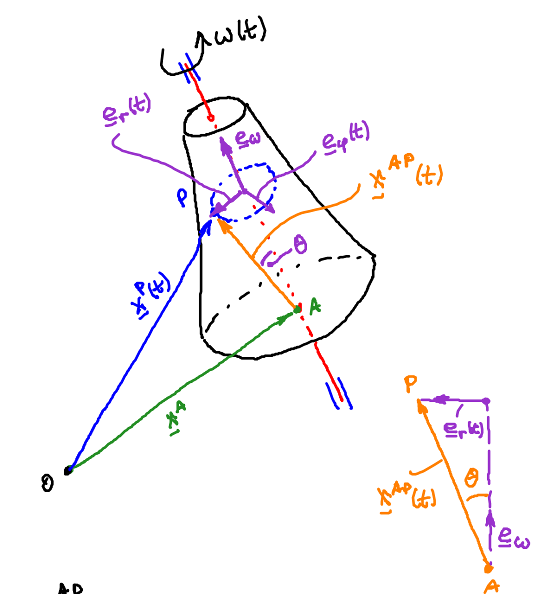


$$\underset{\sim}{v}_s = \frac{d\underset{\sim}{x}_s}{dt}, \quad \underset{\sim}{a}_s = \frac{d^2\underset{\sim}{x}_s}{dt^2}$$

gleich für alle Pkt. des starren Körpers

$$\underset{\sim}{v}^P = \underset{\sim}{v}_s \quad \forall\, P \in \text{starren Körper.}$$

(b) Rotation um eine feste Achse



$$\underset{\sim}{x}^P(t) = \underset{\sim}{x}^A + \underset{\sim}{x}^{AP}(t) \qquad \Big|\ \frac{d}{dt}$$

$$\Rightarrow \underbrace{\frac{d\underset{\sim}{x}^P(t)}{dt}}_{\underset{\sim}{v}^P(t)} = \underset{\sim}{0} + \frac{d\underset{\sim}{x}^{AP}(t)}{dt} \quad \Rightarrow \quad \underset{\sim}{v}^P(t) = \frac{d\underset{\sim}{x}^{AP}(t)}{dt}$$

Eulersche kinematische Gleichung $\quad \underset{\sim}{v}^P(t) = \underset{\sim}{\omega}(t) \times \underset{\sim}{x}^{AP}(t)$

für die Geschwindigkeit

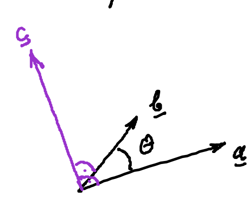
$$\underset{\sim}{\omega}(t) = \omega(t)\,\underset{\sim}{e}_\omega$$

↑ Achsrichtung

$$\underset{\sim}{x}^{AP}(t) = |\underset{\sim}{x}^{AP}| \sin\theta\, \underset{\sim}{e}_r(t) + |\underset{\sim}{x}^{AP}| \cos\theta\, \underset{\sim}{e}_\omega$$

Kreuzprodukt $\quad \underset{\sim}{a} \times \underset{\sim}{b} = \underset{\sim}{c}$

$$|\underset{\sim}{c}| = |\underset{\sim}{a}|\,|\underset{\sim}{b}| \sin\theta$$



$$\frac{d\underline{x}^{AP}(t)}{dt} = |\underline{x}^{AP}| \sin\theta \frac{d\underline{e}_r(t)}{dt} = |\underline{x}^{AP}| \sin\theta \, \omega \, \underline{e}_\varphi(t)$$

$$\frac{d\underline{e}_r}{dt} = \omega(t)\, \underline{e}_\varphi(t)$$

$$\underline{\omega}(t) \times \underline{x}^{AP} = |\underline{\omega}(t)| |\underline{x}^{AP}| \sin\theta \, \underline{e}_\varphi$$

bac cap · Regel

$$\underline{a} \times (\underline{b} \times \underline{c}) = \underline{b}\; \underline{a}\cdot\underline{c} - \underline{c}\; \underline{a}\cdot\underline{b}$$

Nun zur Beschleunigung; wir starten von

$$\underline{v}^P(t) = \underline{\omega}(t) \times \underline{x}^{AP}(t) \quad \text{und diff. nach Zeit}$$

$$\underline{a}^P(t) = \frac{d\underline{v}^P(t)}{dt} = \frac{d}{dt}\left[\underline{\omega}(t) \times \underline{x}^{AP}(t)\right] =$$

$$= \underbrace{\dot{\underline{\omega}}}_{\text{Winkelbeschleunigungsvektor}}(t) \times \underline{x}^{AP}(t) + \underline{\omega}(t) \times \frac{d\underline{x}^{AP}(t)}{dt} =$$

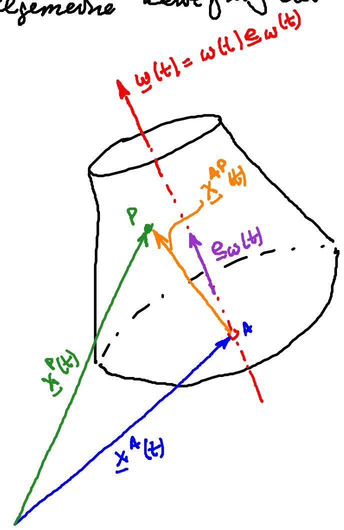
$$\boxed{\underline{a}^P(t) = \dot{\underline{\omega}}(t) \times \underline{x}^{AP}(t) + \underline{\omega}(t) \times \left(\underline{\omega}(t) \times \underline{x}^{AP}(t)\right)}$$

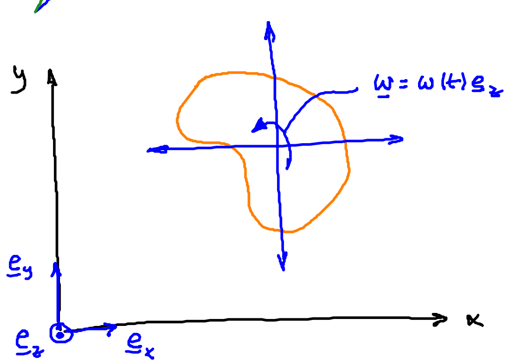
$$\frac{d\underline{x}^{AP}}{dt} = \underline{\omega}(t) \times \underline{x}^{AP}(t) \qquad \underline{a}\cdot\underline{b} = |\underline{a}||\underline{b}|\cos\sphericalangle$$

$$\underline{\omega}(t)\cdot\underline{\omega}(t)$$

$$\Rightarrow \underline{a}^P(t) = \dot{\underline{\omega}}(t) \times \underline{x}^{AP}(t) + \underline{\omega}(t)\; \underline{\omega}(t)\cdot\underline{x}^{AP}(t) - \omega^2(t)\, \underline{x}^{AP}(t)$$

(c) Allgemeine Bewegung des starren Körpers





$$\underline{x}^P(t) = \underline{x}^A(t) + \underline{x}^{AP}(t) \qquad \text{(Orte)}$$

$$\underbrace{\frac{d\underline{x}^P(t)}{dt}}_{\underline{v}^P(t)} = \underbrace{\frac{d\underline{x}^A(t)}{dt}}_{\underline{v}^A(t)} + \underline{\omega}(t) \times \underline{x}^{AP}(t)$$

$$\Rightarrow \underline{v}^P(t) = \underline{v}^A(t) + \underline{\omega}(t) \times \underline{x}^{AP}(t) \qquad \text{(Geschwindigkeiten)}$$

$$\frac{d\underline{v}^P(t)}{dt} = \frac{d\underline{v}^A(t)}{dt} + \dot{\underline{\omega}}(t) \times \underline{x}^{AP}(t) + \underline{\omega}(t) \times \frac{d\underline{x}^{AP}(t)}{dt} =$$

$$\underline{a}^P(t) = \underline{a}^A(t) + \dot{\underline{\omega}}(t) \times \underline{x}^{AP}(t) + \underbrace{\underline{\omega}(t) \times \left(\underline{\omega}(t) \times \underline{x}^{AP}\right)}_{\underline{\omega}(t)\; \underline{x}^{AP}\cdot\underline{\omega}(t) - \omega^2(t)\, \underline{x}^{AP}}$$

(Beschleunigungen)