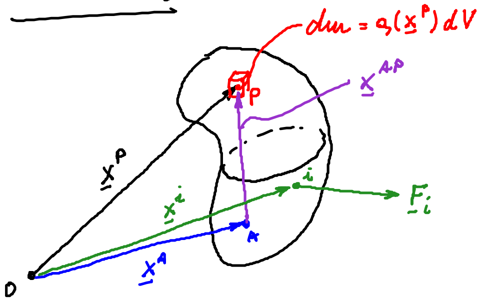
Vorlesung am 22.6.2018

- Wiederholung: Schwerpunktsatz und Drallsatz für 3D/2D
- Beispiele

Wiederholung



kin. Glgn.

$$\underline{x}^P = \underline{x}^A + \underline{x}^{AP}$$
$$\underline{v}^P = \underline{v}^A + \underline{\omega} \times \underline{x}^{AP}$$
$$\underline{a}^P = \underline{a}^A + \dot{\underline{\omega}} \times \underline{x}^{AP} + \underline{\omega} \times (\underline{\omega} \times \underline{x}^{AP})$$
$$= \underline{a}^A + \dot{\underline{\omega}} \times \underline{x}^{AP} + \underline{\omega}\, \underline{\omega} \cdot \underline{x}^{AP} - \underline{x}^{AP} \underline{\omega}^2$$

Impuls: $\underline{P} = \int_m \underline{v}^P\, dm$

Newton: $\dfrac{d\underline{P}}{dt} = \sum_i \underline{F}_i \Rightarrow$

Schwerpunktsatz: $m\, \underline{a}^S = \sum_i \underline{F}_i$

Weg 1

$$\frac{d}{dt}\underline{P} = \frac{d}{dt}\int_m \underline{v}^P\, dm = \frac{d}{dt}\int_m \frac{d\underline{x}^P}{dt}\, dm =$$
$$= \frac{d^2}{dt^2}\int_m \underline{x}^P\, dm = \frac{d^2}{dt^2}\left(m\, \underline{x}^S\right) = m \underbrace{\ddot{\underline{x}}^S}_{\underline{a}^S}$$

$$\underline{x}^S = \frac{1}{m}\int_m \underline{x}^P \overbrace{dm}^{\varrho(\underline{x}^P)\,dV} = \frac{1}{m}\left[\underline{x}^A \int_m dm + \int_m \underline{x}^{AP} dm\right]$$

$$\Rightarrow \int_m \underline{x}^{AP} dm = m\left[\underline{x}^S - \underline{x}^A\right]$$

Weg 2

$$\frac{d}{dt}\underline{P} = \int_m \frac{d^2 \underline{x}^P}{dt^2}\, dm = \int_m \underline{a}^P\, dm =$$
$$= m\, \underline{a}^A + \dot{\underline{\omega}} \times \int_m \underline{x}^{AP} dm + \underline{\omega}\, \underline{\omega} \cdot \int_m \underline{x}^{AP} dm - \underbrace{\underline{\omega}^2}_{\dot{\varphi}^2\ (2D)} \int_m \underline{x}^{AP} dm$$
$$= m\, \underline{a}^A + m\, \dot{\underline{\omega}} \times [\underline{x}^S - \underline{x}^A] + m\, \underline{\omega}\, \underbrace{\underline{\omega} \cdot [\underline{x}^S - \underline{x}^A]}_{=0\ (2D)} - m\, \underline{\omega}^2 [\underline{x}^S - \underline{x}^A]$$
$$= \sum_i \underline{F}_i \qquad \text{falls } A \to S$$

$m\, \underline{a}^S = \sum_i \underline{F}_i \to 2D$ $\underline{a}^S = (\ddot{x}^S, \ddot{y}^S, 0)$

$\underline{\omega} = (0,0,\omega) = (0,0,\dot{\varphi})$ $\underline{x}^S = (x^S, y^S, 0)$ $\underline{x}^A = (x^A, y^A, 0)$ $\underline{F}_i = (F_{ix}, F_{iy}, 0)$ $\underline{a}^A = (\ddot{x}^A, \ddot{y}^A, 0)$

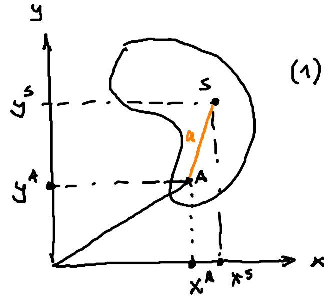
$$m \begin{vmatrix} \underline{e}_x & \underline{e}_y & \underline{e}_z \\ 0 & 0 & \ddot{\varphi} \\ x^S - x^A & y^S - y^A & 0 \end{vmatrix} = m \begin{bmatrix} -\ddot{\varphi}(y^S - y^A) \\ \ddot{\varphi}(x^S - x^A) \\ 0 \end{bmatrix}$$

Drallsatz

$$\frac{d\underline{L}^{(0)}}{dt} = \left[\underline{\underline{\Theta}}^{(A)} \cdot \underline{\omega}\right]^{\cdot} + \underline{x}^A \times \left[\dot{\underline{\omega}} \times m(\underline{x}^S - \underline{x}^A)\right] + \underline{x}^A \times \left[\underline{\omega} \times (\underline{\omega} \times m(\underline{x}^S - \underline{x}^A))\right]$$
$$= \sum_i \underline{x}_i \times \underline{F}^i + \sum_j \underline{M}_j$$

$2D = (x^S, y^S, 0)$

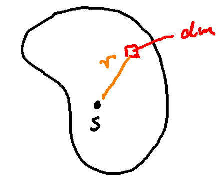
2D **Drallsatz:** $-m y^S \ddot{x}^A + m x^S \ddot{y}^A + y^A m \ddot{\varphi}\,[y^S - y^A] + \Theta^{(A)}\ddot{\varphi} + m\dot{\varphi}^2 y^A (x^S - x^A) - m\dot{\varphi}^2 x^A (y^S - y^A) + x^A m \ddot{\varphi}\,[x^S - x^A] = \sum_i M_i^{(0)}$ (Drallsatz mit Aufpunkt 0)



(1) $\Theta^{(A)}\ddot{\varphi} - m\ddot{\varphi} a^2 - \left(\sum_i F_{ix}\right)(y^S - y^A) + \left(\sum_i F_{iy}\right)(x^S - x^A) = \sum_i M_i^{(A)}$

(Drallsatz mit Aufpunkt A)

A → S : $\boxed{\Theta^{(S)}\ddot{\varphi} = \sum_i M_i^{(S)}}$ $\qquad \Theta^{(S)} = \int_m r^2\, dm$



Was ist, wenn A ein Festlager ist? $\ddot{x}^A = 0$, $\ddot{y}^A = 0$

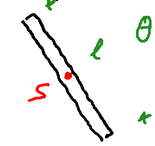
Kraftgleichungen

$$m\ddot{x}^A - m\ddot{\varphi}[y^S - y^A] - m\dot{\varphi}^2[x^S - x^A] = \sum_i F_{ix}$$

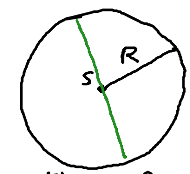
$$m\ddot{y}^A + m\ddot{\varphi}[x^S - x^A] - m\dot{\varphi}^2[y^S - y^A] = \sum_i F_{iy}$$

A → S ⇒ $\boxed{m\ddot{x}^S = \sum_i F_{ix}\,, \quad m\ddot{y}^S = \sum_i F_{iy}}$

Beispiele



$\Theta^{(S)} = \frac{m l^2}{12}$



$\Theta^{(S)} = \frac{m}{2} R^2$

Zurück zur Frage: Was ist, wenn A ein Festlager wird: $x^A = const$, $y^A = const$; $\ddot{x}^A = 0$, $\ddot{y}^A = 0$

Kraftgleichungen:

$$-m\ddot{\varphi}[y^S - y^A] - m\dot{\varphi}^2[x^S - x^A] = \sum_i F_{ix}$$

$$m\ddot{\varphi}[x^S - x^A] - m\dot{\varphi}^2[y^S - y^A] = \sum_i F_{iy}$$

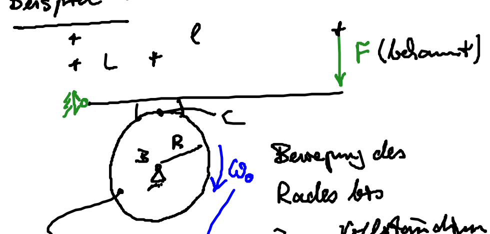
Einsetzen in Drallsatz (1):

$$\Theta^{(A)}\ddot{\varphi} - \cancel{m\ddot{\varphi} a^2} + m\ddot{\varphi}[y^S - y^A]^2 + \cancel{m\dot{\varphi}^2[x^S - x^A][y^S - y^A]} + m\ddot{\varphi}[x^S - x^A]^2 - \cancel{m\dot{\varphi}^2[y^S - y^A][x^S - x^A]} = \sum_i M_i^{(A)}$$

$$m\ddot{\varphi}\left[(x^S - x^A)^2 + (y^S - y^A)^2\right] \longrightarrow a^2$$

⇒ $\boxed{\Theta^{(A)}\ddot{\varphi} = \sum_i M_i^{(A)}}$ Festlager

Beispiel 1

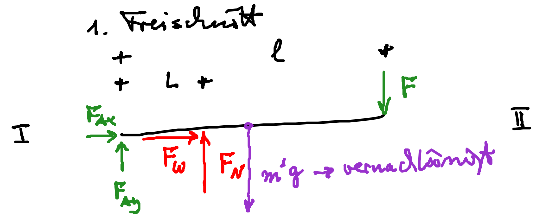


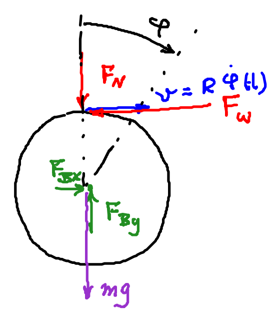
$\Theta^{(B)} = \frac{m}{2} R^2$

Bewegung des Rades bis zum vollständigen Abbremsen?

$\dot{\varphi}(t=0) = \omega_0$

1. Freischnitt





$\sum M^{(A)} = 0$: $\quad F_N L - F l = 0$

⇒ $F_N = \frac{l}{L} F$

Coulomb für Gleitreibung

$F_W = \mu F_N = \mu \frac{l}{L} F$

$-\Theta^{(B)}\ddot{\varphi} = + F_W R$

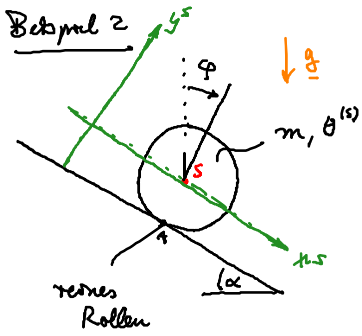
⇒ $\ddot{\varphi} = -\frac{\mu l}{\Theta^{(B)} L} R F$

$$\omega(t): \quad \dot\varphi(t) = -\frac{\mu \ell}{\theta^{(B)} L} RFt + \dot\varphi(0)$$

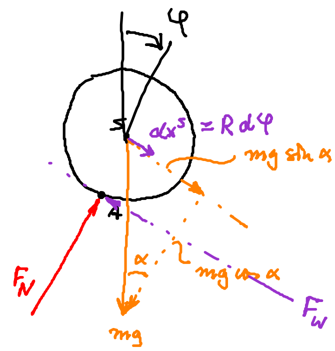
$$= \omega_0 - \frac{\mu \ell}{\theta^{(B)} L} RFt$$

$$\varphi(t) = \omega_0 t - \frac{\mu \ell}{2\theta^{(B)} L} RFt^2$$

Beispiel 2



1) Freischnitt



2)
$$m\ddot{x}^s = mg\sin\alpha - F_W \quad (1)$$
$$m\ddot{y}^s = F_N - mg\cos\alpha \quad (2)$$
$$+\theta^{(s)}\ddot\varphi = +R F_W \ (3), \quad \theta^{(s)} = \frac{m}{2}R^2$$

3) Unbekannte zählen, x^s, y^s, F_W, F_N, φ 5 Stück

4) kinematische Beziehungen $\quad \Rightarrow \ddot\varphi = \frac{\ddot{x}^s}{R}$

Rollbedingung $\quad dx^s = R\,d\varphi \Rightarrow \ddot{x}^s = R\ddot\varphi \quad (4)$

Feste Boden $\quad y^{SA} = const \Rightarrow dy^s = 0 \quad (5)$

$\ddot{y}^s = 0$

$$(1) \Rightarrow m\ddot{x}_s = mg\sin\alpha - \frac{\frac{m}{2}R^{\cancel{2}}}{\cancel{R}}\ddot\varphi = mg\sin\alpha - \frac{m}{2}\ddot{x}_s$$

$$\Rightarrow \quad \frac{3}{2}\cancel{m}\,\ddot{x}_s = \cancel{m}\,g\sin\alpha \quad \Rightarrow \quad \ddot{x}_s = \frac{2}{3}g\sin\alpha$$

Achtung bei Rutschen : $\ddot{x}_s = g\sin\alpha$

$$(2) \Rightarrow \quad F_N = mg\cos\alpha$$