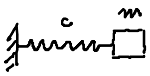
Vorlesung am 6.7.2018

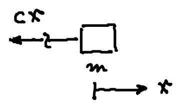
- Wiederholung: Schwingungsdgl. ohne Dämpfung
- Lösungsansatz mit exp-Funktion
- Energiesatz
- Federkonstanten

- Schwingung mit Coulomb dämpfung

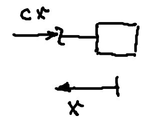
Wiederholung



Freischnitt a



Freischnitt



$$m\ddot{x} = -cx$$

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$$\ddot{x} + \underbrace{\frac{c}{m}}_{\omega^2} x = 0 \qquad (1)$$

Eigenfrequenz $\omega = \sqrt{\frac{c}{m}} = \frac{2\pi}{T}$ ← Schwingungsdauer

$$T = 2\pi\sqrt{\frac{m}{c}}$$

Lösung der Dgl. (1) $\quad \ddot{x} + \omega^2 x = 0$, gesucht $x(t)$

1) Fußgängermethode

$$x(t) = A\sin(\omega t) + B\cos(\omega t)$$

$$\dot{x}(t) = \omega\left[A\cos(\omega t) - B\sin(\omega t)\right]$$

AB : 1) $x(t=0) = x_0 = A\cdot 0 + B\cdot 1 \Rightarrow B = x_0$

2) $\dot{x}(t=0) = v_0 = \omega\left[A\cdot 1 - B\cdot 0\right] \Rightarrow A = \frac{v_0}{\omega}$

$$\Rightarrow x(t) = \frac{v_0}{\omega}\sin(\omega t) + x_0\cos(\omega t) \qquad \text{harmonische Schwingung}$$

Euler $\exp(i\alpha) = \cos\alpha + i\sin\alpha$

$\exp(-i\alpha) = \cos\alpha - i\sin\alpha$

2) Exponentialansatz

$x(t) = A\exp(\beta t)$

$\dot{x}(t) = \beta A\exp(\beta t)$

$\ddot{x}(t) = \beta^2 A\exp(\beta t) = \beta^2 x(t)$

In (1) : $\left[\beta^2 + \omega^2\right]\underbrace{x(t)}_{A\exp(\beta t)} = 0$

$\Rightarrow \beta^2 = -\omega^2 \Rightarrow \beta = \pm i\omega \Rightarrow x(t) = A_1\exp(i\omega t) + A_2\exp(-i\omega t)$

$$= \underbrace{(A_1 + A_2)}_{B}\cos(\omega t) + i\underbrace{(A_1 - A_2)}_{\hat{=}A}\sin(\omega t)$$

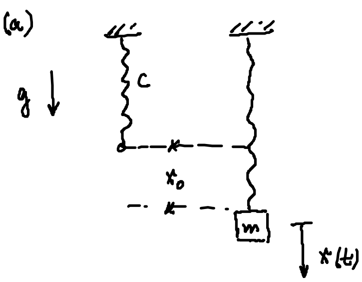
$A_1 = A_1^1 + iA_1^2$

$A_2 = A_2^1 + iA_2^2$

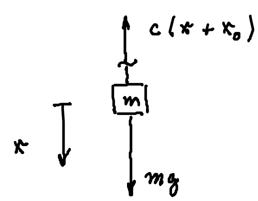
$$A_1^2 = -A_2^2, \quad A_1^1 = A_2^1$$

Die Dgl. des harmonischen 1D-Schwingers in "Verkleidung"

(a)



Freischnitt



Gleichung:

$$m\ddot{x} = -c(x + x_0) + mg$$

Statische Ruhelage $x_{st} \Rightarrow \ddot{x}_{st} = 0$

$$0 = -c(x_{st} + x_0) + mg$$

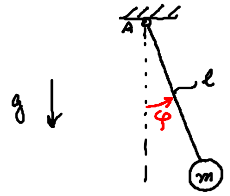
$$x_{st} = \frac{mg}{c} - x_0$$

Def.: $\tilde{x} = x - x_{st} \Rightarrow \ddot{\tilde{x}} = \ddot{x}$

$$\Rightarrow m\ddot{\tilde{x}} = -c(\tilde{x} + \underbrace{x_{st}}_{\frac{mg}{c} - x_0} + x_0) + mg =$$

$$= -c\tilde{x} - \cancel{mg} + \cancel{mg}$$

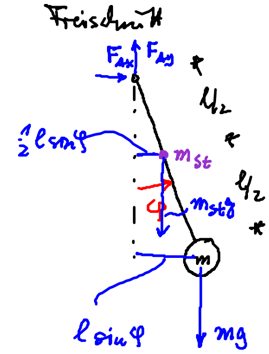
$$\Rightarrow \ddot{\tilde{x}} + \frac{c}{m}\tilde{x} = 0$$

(b) 

(1) Faden masselos

(2) Stange mit Masse m_{St}

Freischnitt



Drallsatz

$$+\ \Theta^{(A)}\ddot{\varphi} = -mg\ell\sin\varphi$$

(1) $\Theta^{(A)} = m\ell^2$

(2) $\Theta^{(A)} = m\ell^2 + \frac{m_{St}}{3}\ell^2$

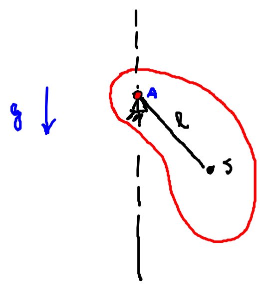
(1) : $\ddot{\varphi}\cancel{m}\ell^{\cancel{2}} = -\cancel{m}g\cancel{\ell}\sin\varphi$

$$\Rightarrow \ddot{\varphi} + \underbrace{\frac{g}{\ell}}_{\omega^2}\sin\varphi = 0$$

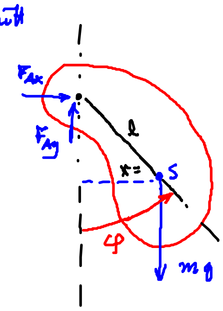
$\sin\varphi \approx \varphi$ für kleine Auslenkungen

$$\ddot{\varphi} + \omega^2\varphi = 0$$

(2) $\left(m + \frac{m_{St}}{3}\right)\ell^2\ddot{\varphi} = -mg\ell\sin\varphi - m_{St}g\frac{\ell}{2}\sin\varphi$

(c) 

Freischnitt

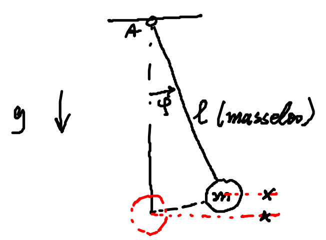


Drallsatz

$$+\ \Theta^{(A)}\ddot{\varphi} = -mg\ell\sin\varphi$$

$$\ddot{\varphi} + \underbrace{\frac{mg\ell}{\Theta^{(A)}}}_{=\omega^2}\underbrace{\sin\varphi}_{\approx\varphi} = 0$$

Energiesatz am Beispiel des Fadenpendels



$$\omega^2 = \frac{g}{\ell}$$

Energiesatz:

$$E^{kin}(t) + E^{pot}(t) = E^{kin}(0) + E^{pot}(0)$$

$$E^{kin}(t) = \frac{m}{2}(v_\varphi)^2 = \frac{m}{2}\left(\ell\dot{\varphi}(t)\right)^2$$

$\cos\varphi \approx 1 - \frac{\varphi^2}{2}$

$$E^{pot}(t) = mg(\ell - \ell\cos\varphi) = mg\ell\,(1-\cos\varphi(t)) \approx mg\ell\frac{\varphi^2}{2}$$

$$E^{kin}(0) = 0$$

$$E^{pot}(0) = mg\ell\frac{\varphi_0^2}{2} = E_0$$

Lösung der Bewegung: $\varphi(t) = \varphi_0\cos(\omega t)$

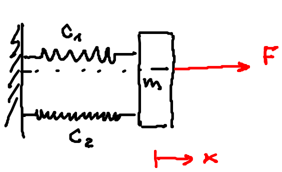
$$E^{kin}(t) = \frac{m\ell^2}{2}\dot{\varphi}^2(t) = \ldots = \frac{E_0}{2}[1-\cos(2\omega t)]$$

$$E^{pot}(t) = \frac{m\ell^2}{2}\varphi^2(t) = \ldots = \frac{E_0}{2}[1+\cos(2\omega t)]$$

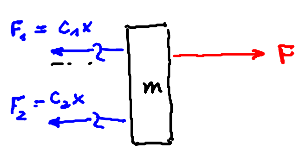
Energie schwingen doppelt so schnell

Ersatzschaltungen für mehrere Systemfedern

(a) Parallelschaltung



Freischnitt

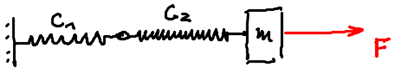


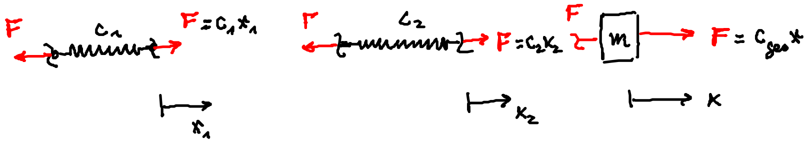
Wie groß ist die Ersatzsteifigkeit c_{ges} als Fkt. von c_1 und c_2

$$F = F_1 + F_2$$

$$c_{ges}x = (c_1 + c_2)x \quad \Rightarrow \quad c_{ges} = c_1 + c_2$$

(b) Serienschaltung





$$x_1 + x_2 = x \quad \Rightarrow \quad \frac{\cancel{F}}{c_1} + \frac{\cancel{F}}{c_2} = \frac{\cancel{F}}{c}$$

$$\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2}$$