Probeklausur 4.6.18

- Homepage erreichbar unter:

  https://www.lkm.tu-berlin.de/menue/studium_und_lehre/lehrangebot/kinematik_und_dynamik_sose_18/

- Lehrevaluation:

  https://befragung.tu-berlin.de/evasys/online.php?p=59EYA

Theorie

1.) $$[E_{pot}] = N \cdot m = \frac{kg \cdot m}{s^2} \cdot m = kg \cdot \frac{m^2}{s^2}$$

$$[p] = kg \cdot \frac{m}{s}$$

$$[c] = \frac{N}{m} = \frac{kg \cdot m}{s^2 m} = \frac{kg}{s^2}$$

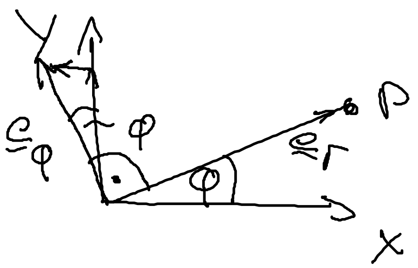
$$[\ddot{\varphi}] = \frac{1}{s^2}$$

2.) $S_L = S_H \cdot e^{\mu \alpha}$

$F_1 = F_2 \cdot e^{\mu_0 \alpha}$ (Euler - Eytelwein'sche Seilreibung)

3.) $\underline{x} = R \cdot \underline{e}_r$

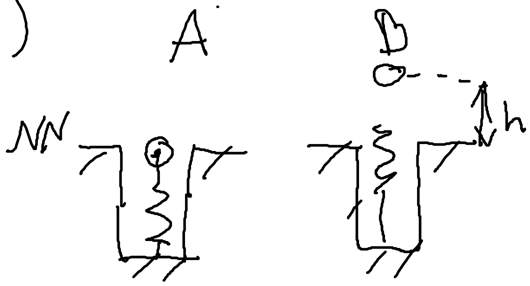
$$\underline{V} = \underbrace{\cancel{\dot{R}}}_{=0}\,\underline{e}_r + R\cdot\dot{\underline{e}}_r = R\cdot\dot{\varphi}\,\underline{e}_\varphi = R\omega\,\underline{e}_\varphi$$



$$\underline{e}_\varphi = -\sin\varphi\,\underline{e}_x + \cos\varphi\,\underline{e}_y$$

$$\underline{V}(\varphi) = R\omega\left(-\sin\varphi\,\underline{e}_x + \cos\varphi\,\underline{e}_y\right)$$

4.)



$$E_A^{pot} + E_A^{kin} = E_B^{pot} + E_B^{kin}$$

$$\frac{1}{2}c\Delta x^2 + 0 = mgh + 0$$

$$\Leftrightarrow h = \frac{c\Delta x^2}{2mg}$$

5.) $a = \frac{dv}{dt}$ $\quad V = V(X(t))$ $\quad V \circ X$

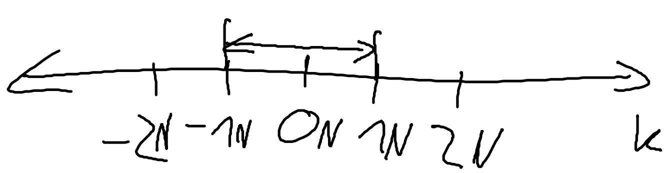
$$a = \frac{dv(x(t))}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{dv}{dx}\cdot v = Ae^{Cx}\cdot C\cdot Ae^{Cx}$$

$$= A^2\cdot Ce^{2Cx} = a(x) \quad \text{(Kettenregel)}$$

6.) $F_x = -\frac{\delta E_{pot}}{\delta x} = m\omega^2 x$ $\quad F_y = -\frac{\delta E_{pot}}{\delta y} = -mg$

7.) $F_N = 2N$ $\quad F_W = \mu_H F_N = 0{,}5\cdot 2N = 1N$

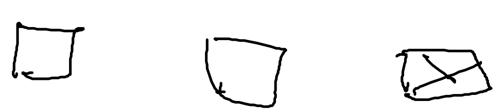
$|k| \leq 1N$



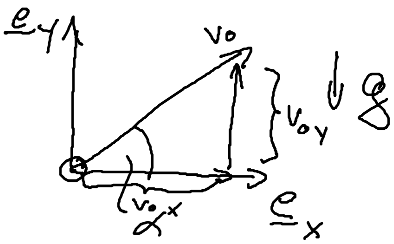
1. $k \leq 1N$
2. $-k \leq 1N$

$k \geq -1N$

8.) $a = \ddot{x}$



Rechenaufgabe 1)

a) 

geg: v_0, α

$$\underline{a}(t) = -g\,\underline{e}_y$$

$$\underline{v}(t) = \int \underline{a}(t)\,dt = -gt\,\underline{e}_y + \underline{v}_0 = -gt\cdot\underline{e}_y + \cos\alpha\, v_0\,\underline{e}_x + \sin\alpha\, v_0\,\underline{e}_y$$

$$= \cos\alpha\, v_0\,\underline{e}_x + (\sin\alpha\, v_0 - gt)\,\underline{e}_y$$

$$\underline{x}(t) = \int \underline{v}(t)\,dt = \cos\alpha\, v_0 t\,\underline{e}_x + \left(\sin\alpha\, v_0 t - g\frac{t^2}{2}\right)\underline{e}_y \;//$$

$$x(t) = v_0\cos\alpha\cdot t$$

$$y(t) = v_0\sin\alpha\, t - g\frac{t^2}{2}$$

b) ges: t_e

$$x(t_e) \overset{!}{=} d$$

$$d = v_0\cos\alpha\cdot t_e$$

$$\Leftrightarrow t_e = \frac{d}{v_0\cos\alpha} \;//$$

c) ges: v_0, sodass der Ball bei t_e die Höhe h erreich

$y(t_e) \overset{!}{=} h$

$$h = v_0 \cdot \sin\alpha \cdot t_e - \frac{g t_e^2}{2}$$

$$h = \cancel{v_0} \cdot \sin\alpha \cdot \frac{d}{\cancel{v_0}\cos\alpha} - g \cdot \frac{d^2}{2v_0^2 \cdot \cos^2\alpha} \qquad \Big| \cdot (-1) \; + \sin\alpha \frac{d}{\cos\alpha}$$

$$\Leftrightarrow \frac{\sin\alpha}{\cos\alpha} \cdot d - h = g \cdot \frac{d^2}{2v_0^2 \cdot \cos^2\alpha} \qquad \Big| \cdot v_0^2$$

$$v_0^2 \left(\frac{\sin\alpha}{\cos\alpha} \cdot d - h\right) = g \cdot \frac{d^2}{2\cos^2\alpha} \qquad \Big| : (\,) \quad \sqrt{}$$

$$v_0 = \sqrt{\frac{g d^2}{2(d\sin\alpha\cos\alpha - h\cos^2\alpha)}}$$

d) ges: d_{min}, sodass v_0 gerade noch reell ist

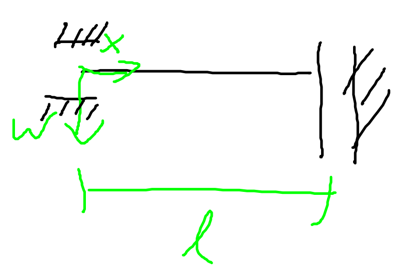
$v_0 \in \mathbb{R} \Rightarrow$ Wurzel nicht negativ

$$d\sin\alpha\cos\alpha - h\cos^2\alpha > 0$$

$$d\sin\alpha\,\cancel{\cos\alpha} > h\cos^{\cancel{2}}\alpha \qquad \alpha \in [0, \tfrac{\pi}{2}]$$

$$d > \frac{h}{\tan\alpha} = h \cdot \cot\alpha$$

2)



a) (I) $w(x=0) = 0$

(II) $w'(x=0) = 0$

(III) $w'(x=l)=0$

(IV) $Q(x=l)=0 \Leftrightarrow w'''(x=l)=0$

$$w'(x) = -A\alpha\sin(\alpha x) + B\alpha\cos(\alpha x) + C\alpha$$

$$w''(x) = -A\alpha^2\cos(\alpha x) - B\alpha^2\sin(\alpha x)$$

$$w'''(x) = A\alpha^3\sin(\alpha x) - B\alpha^3\cos(\alpha x)$$

(I) $A + D = 0$

(II) $B\cancel{\alpha} + C\cancel{\alpha} = 0 \quad | \ \alpha = 0 = F$

(III) $-A\cancel{\alpha}\sin(\alpha l) + B\cancel{\alpha}\cos(\alpha l) + C\cancel{\alpha} = 0$

(IV) $A\cancel{\alpha^3}\sin(\alpha l) - B\cancel{\alpha^3}\cos(\alpha l) = 0$

b) Trick für Eigenwertgleichung:

$$\begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ -\sin(\alpha l) & \cos(\alpha l) & 1 & 0 \\ \sin(\alpha l) & -\cos(\alpha l) & 0 & 0 \end{bmatrix} \end{array} \cdot \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \underline{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{\underline{M}} \cdot \underline{v} = \underline{0} \Leftrightarrow \underline{V} = \underline{0} \ \vee \ \det(\underline{\underline{M}}) = 0$$

$$\det(\underline{\underline{M}}) = -\underline{\underline{\sin(\alpha l) = 0}}$$

c)
$$\sin(\alpha l) = 0$$
$$\Leftrightarrow \alpha l = \pi n \quad , n \in \mathbb{Z}$$
$$\Leftrightarrow \alpha = \frac{\pi n}{l} = \sqrt{\frac{F}{EI}}$$

F_{krit} bei $n=1$

$$\Rightarrow \frac{\pi}{l} = \sqrt{\frac{F_{krit}}{EI}} \Leftrightarrow F_{krit} = \underline{\underline{\frac{\pi^2 EI}{l^2}}}$$

$$\frac{\frac{N}{\cancel{m^2}} \cancel{m^4}^{2}}{\cancel{m^2}}$$

d) 1. Eigenform ($n = 1 \Leftrightarrow \alpha l = \pi$)

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$\underline{IV}$ $\underline{\underline{B = 0}}$

$\underline{II}$ $B + C = 0 \Leftrightarrow \underline{\underline{C = 0}}$

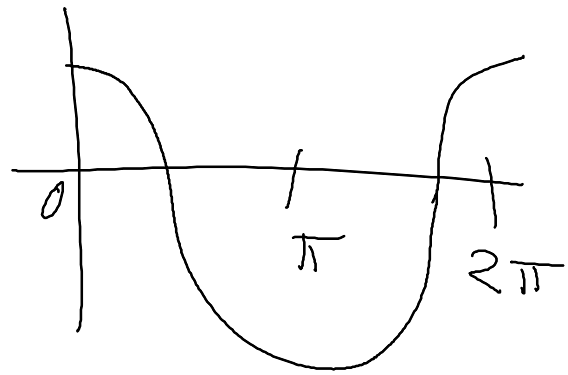
$$\text{I} \quad A + D = 0 \iff A = -D = w^*$$

$$w_1(x) = w^*\left(\cos\left(\frac{\pi}{l}x\right) - 1\right)$$

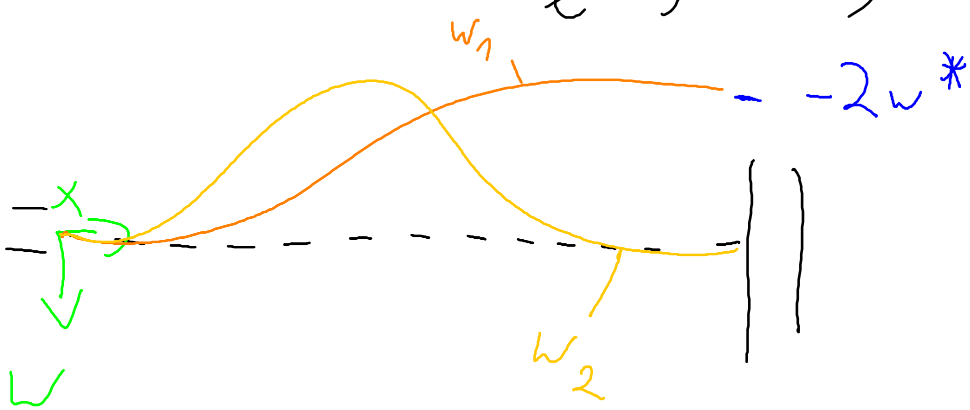
2\. Eigenform $(n=2) \Rightarrow \alpha l = 2\pi$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad C = 0$$

$$\Rightarrow B = 0$$



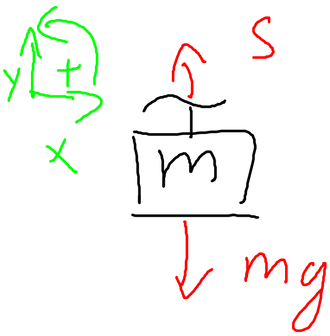
$$w_2(x) = w^*\left(\cos\left(\frac{2\pi}{l}x\right) - 1\right)$$



3)

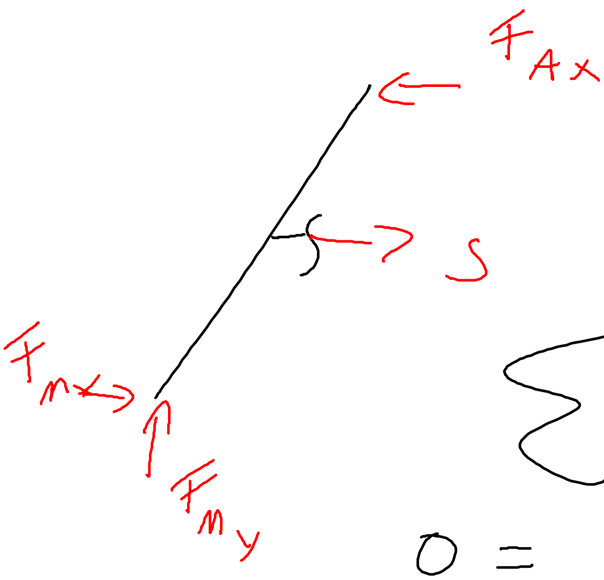
a) b) c)

FS 1



$$\sum F_y \overset{!}{=} 0 \Leftrightarrow \underline{\underline{S = mg}}$$

FS 2



$$\sum F_x \overset{!}{=} 0 \Leftrightarrow F_{Mx} + S - F_{Ax} = 0$$

$$\Leftrightarrow \underline{\underline{F_{Mx}}} = F_{Ax} - S = \underline{\underline{-\frac{mg}{2}}}$$

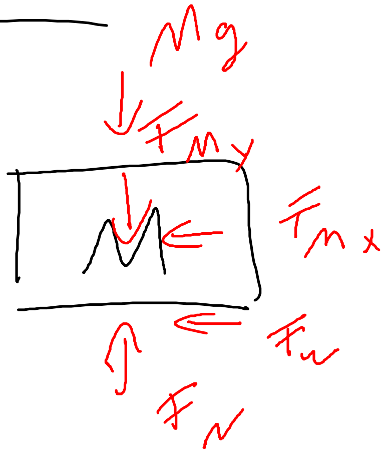
$$\sum F_y \overset{!}{=} 0 \Leftrightarrow \underline{\underline{F_{My} = 0}}$$

$$\sum M^{(M)} \overset{!}{=} 0 \Leftrightarrow$$

$$0 = -S \frac{\cancel{l}}{2} \cancel{\sin(\beta)} + F_{Ax} \cancel{l} \cancel{\sin(\beta)}$$

$$\Leftrightarrow \underline{\underline{F_{Ax} = \frac{S}{2} = \frac{mg}{2}}}$$

FS 3



$$\sum F_y \overset{!}{=} 0$$

$$\Leftrightarrow 0 = -Mg + F_N \Leftrightarrow \underline{\underline{F_N = Mg}}$$

$$\underline{\underline{F_W = \mu Mg}}$$

$$\sum F_x \stackrel{!}{=} 0 \Leftrightarrow -F_{nx} - F_w = 0$$

$$\Leftrightarrow F_w = -F_{nx} = \frac{m\cancel{g}}{2} = \mu M \cancel{g}$$

$$\Leftrightarrow \underline{\underline{M = \frac{m}{2\mu}}}$$

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