Material plasticity to track the elastic anisotropy at finite strains

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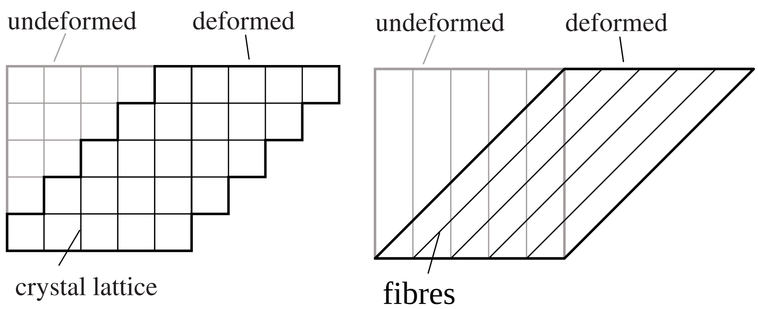
We present a phenomenological finite elasto-plasticity theory that includes the evolution of the elastic anisotropy. The theory is called Material Plasticity after [1] and [2], assuming that the axes of anisotropy deform as material line elements. It is applicable to fiber-reinforced materials. The main feature is the different evolutions of the stiffness tetrad and of the stress-free placement. For this purpose we extend the well known isomorphy concept (see [3]).

To identify and compare the stiffness tetrads before and after large plastic deformations a representative volume element (RVE) [4] with a fiberous microstructure is used. Uni-, bi- and and tridirectionally reinforced samples are considered. On the microscale a standard elastic-plastic material model is used. After calculating the effective stiffnesses of the different material samples we investigate their evolution during different deformations. It turns out that it is possible to denote the change of the stiffness with sufficient accuracy using one additional second order tensor. After investigating the change in the stiffness tetrads we finally propose an analytical evolution equation for this tensor.

1 Introduction

In crystal plasticity, the crystal structure is preserved during plastic deformations, which gives rise to a constant elastic reference law. However, this is a very special case. In general, the microstructure and with it the elastic properties evolve during plastic deformations. The intention of material plasticity is a model for a heterogenous material having a microstructure which deforms together with the material under plastic deformations (see Figure 1(b)). The fibers are directly fixed to the material. For such a theory, the constant elastic reference law has to be replaced by a more suitable assumption.



(a) Crystal plasticity (b) Material plasticity

Fig. 1 Introduction of a material plasticity theory with fibers deforming together with the material on the right and the classical crystal plasticity theory with the orientation of the crystal lattice remaining the same with respect to the shear direction and slip plane normal while the material deforms on the left.

The starting point is the isomorphy concept used for crystal plasticity (see Equation 1_1).

$$\overset{\text{2PK}}{\boldsymbol{T}} = {}^1\!/_2 \left(\boldsymbol{P}_C * \mathbb{K}_0\right) : \left(\boldsymbol{C} - \boldsymbol{P}_C^{-T} * \boldsymbol{C}_{u0}\right) \qquad \overset{\text{2PK}}{\boldsymbol{T}} = {}^1\!/_2 \left(\boldsymbol{P}_K * \mathbb{K}_0\right) : \left(\boldsymbol{C} - \boldsymbol{P}_C^{-T} * \boldsymbol{C}_{u0}\right) \tag{1}$$

With the Rayleigh product $\boldsymbol{P} * \mathbb{C} = \boldsymbol{P} * (C_{ij}\boldsymbol{e}_i \otimes \boldsymbol{e}_j) = C_{ij}(\boldsymbol{P}\boldsymbol{e}_i) \otimes (\boldsymbol{P}\boldsymbol{e}_j)$. This theory is generalized by permitting a different evolution of the stiffness tetrad relatively to the evolution of the stress-free placement by replacing $\boldsymbol{P}_C$ in front of $\mathbb{K}_0$ by a second transformation $\boldsymbol{P}_K$. Isomorphy is contained as the special case $\boldsymbol{P}_K = \boldsymbol{P}_C$ (see Equation 1_2). The main question to verify is whether this is sufficient to approximate the evolution of the stiffness tetrad. From a point of view of representation theory, this is not the case. However, from the engineers point of view, the accuracy may be sufficient, and the approach relatively simple. To quantify this, a distance measure is defined

$$d = ||\mathbb{K}_1 - \boldsymbol{P}_K * \mathbb{K}_0|| \,/\, ||\boldsymbol{P}_K * \mathbb{K}_0|| \,. \tag{2}$$

2 Microscale

We use an RVE with unidirectional fiber reinforcement having effectively transversal isotropic material symmetry, a bi-directional fiber alignment with effectively tetragonal symmetry, and a tri-directional material with effectively cubic symmetry. Locally, an isotropic elastic-plastic material model is used with three parameters E, ν, σ_F for both the matrix and

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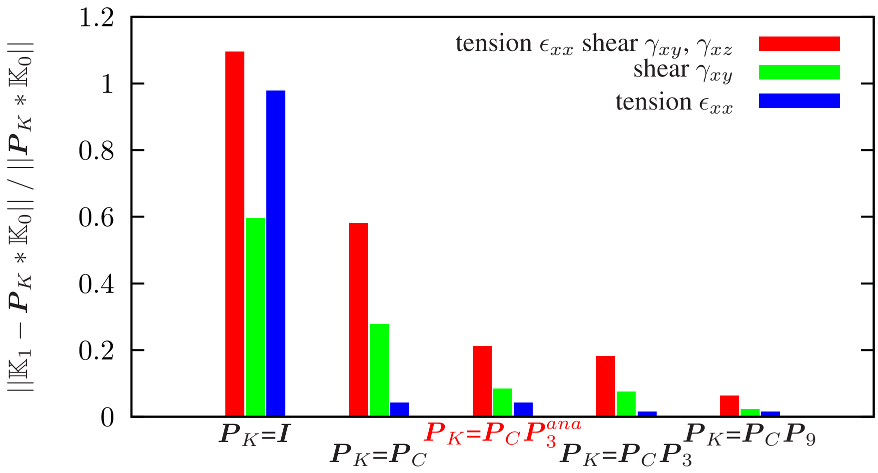

Fig. 2 Deviation of the measured stiffness tetrad from the respective modelling approach for the tri-directional reinforcement. The choice $\boldsymbol{P}_K = \boldsymbol{I}$ is clearly the worst, followed by elastic isomorphy $\boldsymbol{P}_K = \boldsymbol{P}_C$. With three degrees of freedom a significant improvement is reached. The analytical approach is almost as good as the best numerical fit.

the fiber material, where the fiber is much stiffer and has a higher yield strength. The elastic behaviour is modelled by the St.-Venant-Kirchhoff law, the von Mises yield limit and an associative flow rule are used. The difference quotient in six test calculations is used to determine the effective stiffnesses $\mathbb{K}_0$ before and $\mathbb{K}_1$ after a large plastic deformation.

3 Macroscale

In Fig. 2 some choices for the transformation tensor $\boldsymbol{P}_K$ are listed for the tri-directionally reinforced material. The approaches are $\boldsymbol{P}_K = \boldsymbol{I}$ to not transform $\boldsymbol{K}_0$ at all, $\boldsymbol{P}_K = \boldsymbol{P}_C$ assuming elastic isomorphy, $\boldsymbol{P}_K = \boldsymbol{P}_C\boldsymbol{P}_9$ with $\boldsymbol{P}_9$ being the numerically best fit for all nine components of $\boldsymbol{P}_K$, and $\boldsymbol{P}_K = \boldsymbol{P}_C\boldsymbol{P}_3$ with $\boldsymbol{P}_3$ being the numerically best fit using only the three main diagonal entries of the transformation tensor. This limits us to materials with perpendicular fiber directions. Firstly, it turns out that it is possible to approximate the evolution of the stiffness tetrad with sufficient accuracy using one second order tensor. Secondly, we find the best compromise between accuracy and effort with $\boldsymbol{P}_K$=$\boldsymbol{P}_C\boldsymbol{P}_3$. To derive an analytical evolution equation for $\dot{\boldsymbol{P}}_3$ we use the pullback $\boldsymbol{L}^{ref} = \boldsymbol{F}^{-1}\boldsymbol{L}\boldsymbol{F}$ of the velocity gradient $\boldsymbol{L} = \dot{\boldsymbol{F}}\boldsymbol{F}^{-1}$, motivated by the fact that the fiber directions are constant in the reference placement.

$$\dot{\boldsymbol{P}}_3 = \begin{bmatrix} \dot{P}_{11} & 0 & 0 \\ 0 & \dot{P}_{22} & 0 \\ 0 & 0 & \dot{P}_{33} \end{bmatrix} \qquad \dot{P}_{ii} = k_i\sqrt{L^{ref\,2}_{i+1\;i} + L^{ref\,2}_{i+2\;i}} \qquad \boldsymbol{L}^{ref} = \boldsymbol{F}^{-1}\dot{\boldsymbol{F}} \tag{3}$$

The coefficients k_i depend on the material parameters and on the volume fractions of the fibers in direction i ($k_i = 0$ if there is no fiber in direction i, the indices $i + 1$ and $i + 2$ are taken modulo 3). To determine the coefficients we use RVE calculations for the three characteristic shear tests γ_{21} (k_1), γ_{12} (k_2) and γ_{13} (k_3) and take the i'th entry in the numerically best $\boldsymbol{P}_3$ for k_i.

4 Summary

We use RVEs for modeling a uni-directonal, bi-directional and tri-directional material (all having different material symmetries) using an elastic-plastic material model for large deformations. We made numerical calculations to determine the effective stiffness of the inhomogenous materials after large deformations and it turned out that it is possible to predict the evolution of the stiffness tetrads during large deformations for a class of fiber reinforced materials having fiber angles of 90° with sufficient accuracy using one second order tensor $\boldsymbol{P}_K = \boldsymbol{P}_C\boldsymbol{P}_3$ within the introduced equation for a material plasticity theory.

References

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